"Model"

HW#1 – Math 10260

Section 5.1

(62) $MC = 6q^2 - 4q + 8$, where q = the number of units produced & $C(1) = 500$. What is the total cost of producing 20 units?

Step 1: Take the indefinite integral.

$$MC = C'(q) = 6q^2 - 4q + 8$$

$$C(q) = \int (6q^2 - 4q + 8)\,dq = 2q^3 - 2q^2 + 8q + c$$

Step 2: Use the intial condition to find c.

$$C(1) = 2(1)^3 - 2(1)^2 + 8(1) + c = 500$$

$$8 + c = 500$$

$$c = 492$$

Step3: Compute the cost of producing 20 units.

$$C(20) = 2(20)^3 - 2(20)^2 + 8(20) + 492$$

$$\boxed{= \$15{,}852}$$

Section 5.2

(18) Find the indefinite integral of $\int t^3\sqrt{t^4+1}\,dt$.

$u = t^4 + 1$ and $du = 4t^3\,dt$

$\frac{1}{4}du = t^3\,dt$

Step1: Use substitution to find the indefinite integral.

$$\int t^3\sqrt{t^4+1}\,dt = \int \sqrt{u}\cdot \tfrac{1}{4}du$$

$$= \tfrac{1}{4}\int u^{1/2}\,du$$

$$= \tfrac{1}{6}u^{3/2} + c$$

$$\boxed{= \tfrac{1}{6}(t^4+1)^{3/2} + c}$$

Step2: Check by differentiating.

$$\frac{d}{dt}\left(\tfrac{1}{6}(t^4+1)^{3/2} + c\right) = \tfrac{1}{6}\cdot\tfrac{3}{2}(t^4+1)^{1/2}\cdot 4t^3 + 0 \overset{\checkmark}{=} t^3\sqrt{t^4+1}$$

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HW # 1

Sec. 5.2 cont.

(27) Find the indefinite integral of $\int \frac{dt}{t \ln t}$
Let $u = \ln t$ and $du = \frac{1}{t} dt$

Step 1: Use substitution to find the indefinite integral.

$$\int \frac{dt}{t \ln t} = \int \frac{1}{\ln t} \cdot \frac{1}{t} dt$$
$$= \int \frac{1}{u} du$$
$$= \ln|u| + c$$
$$\boxed{= \ln|\ln t| + c}$$

Step 2: Check by differentiating.

$$\frac{d}{dt}(\ln|\ln t| + c) = \frac{1}{\ln t} \cdot \frac{1}{t} + \cancel{c} \overset{\checkmark}{=} \frac{1}{t \ln t}$$

Section 5.3

(8) Find the integral of $\int t^3 e^t dt$ using integration by parts.
Let: $u = t^3$ $\quad v = e^t$
$du = 3t^2 dt$ $\quad dv = e^t dt$, $\int uv' dx = uv - \int v u' dx$

Step 1: Rearrange terms to give the integration by parts formula.

$$\int t^3 e^t dt = t^3 e^t - 3\int t^2 e^t dt$$

Step 2: Apply integration by parts using:
$u = t^2$ $\quad v = e^t$
$du = 2t\,dt$ $\quad dv = e^t dt$

$$\int t^2 e^t dt = t^2 e t - 2\int t e^t dt$$

Step 3: Apply integration by parts using:
$u = t$ $\quad v = e^t$
$dv = dt$ $\quad dv = e^t dt$

$$\int t e^t dt = t e^t - \int e^t dt = t e^t - e^t + c$$

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HW#1

Sec. 5.3 cont.

(8) Step 4: Combine these steps.

$$\int t^3 e^t dt = t^3 e^t - 3(t^2 e^t - 2te^t + 2e^t) + c$$
$$\boxed{= t^3 e^t - 3t^2 e^t + 6te^t - 6e^t + C}$$

(44) Find the indefinite integral of $\int x^6 \ln x \, dx$

Let: $u = \ln x$ $\quad v = \frac{1}{7}x^7$

$du = \frac{1}{x}dx \quad dv = x^6 dx$, $\int uv' dx = uv - \int vu' dx$

Step 1: Rearrange terms to give the integration by parts formula.

$$\int x^6 \ln x \, dx = \frac{1}{7}x^7 \ln x - \frac{1}{7}\int x^6 dx$$

Step 2: Solve.

$$\int x^6 \ln x \, dx = \frac{1}{7}x^7 \ln x - \frac{1}{7}\int x^6 dx$$
$$= \frac{1}{7}x^7 \ln x - \frac{1}{7} \cdot \frac{1}{7}x^7 + c$$
$$\boxed{= \frac{1}{7}x^7 \ln x - \frac{1}{49}x^7 + C}$$