Name _____ Date _____

Math 10260 Homework 24: Geometric Series and Applications (Section 10.2)

****** Make **double-sided** copy of this and turn in completed work to class. *******

1. For each of the infinite series below take ratio of consecutive terms of the series to check for common ratio. Assuming that the pattern in the series persist determine if it is a geometric series. Check the answer for each series and fill in the blanks where applicable.

(a) (10.2/Q5) Series A: $1 + e^{-1} + e^{-2} + e^{-3} + \cdots$

[ ] Series A is NOT a geometric series.

[ ] Series A is a geometric series with first term _____ and common ratio _____. The sum of the series is _____ (write undefine if divergent).

(b) Series B: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

[ ] Series B is NOT a geometric series.

[ ] Series B is a geometric series with first term _____ and common ratio _____. The sum of the series is _____ (write undefine if divergent).

(c) (10.2/Q4) Series C: $1 - 1 + 1 - 1 + 1 - \cdots$

[ ] Series C is NOT a geometric series.

[ ] Series C is a geometric series with first term _____ and common ratio _____. The sum of the series is _____ (write undefine if divergent).

(d) Series D: $\sum_{k=1}^{\infty} 2^{-3k}$

[ ] Series D is NOT a geometric series.

[ ] Series D is a geometric series with first term _____ and common ratio _____. The sum of the series is _____ (write undefine if divergent).

(e) Series E: $\frac{1}{5} - \frac{1}{5^2} + \frac{1}{5^3} - \frac{1}{5^4} - \cdots$

[ ] Series E is NOT a geometric series.

[ ] Series E is a geometric series with first term _____ and common ratio _____. The sum of the series is _____ (write undefine if divergent).

(f) Series F: $\frac{\pi}{3} + \frac{\pi^2}{9} + \frac{\pi^3}{27} + \frac{\pi^4}{81} + \cdots$

[ ] Series F is NOT a geometric series.

[ ] Series F is a geometric series with first term _____ and common ratio _____. The sum of the series is _____ (write undefine if divergent).

2. Rewrite each of the given infinite (repeated) decimal as an infinite series. By summing a geometric series, represent the given infinite decimal as a simple fraction. Do all your work without a calculator.

(a) (10.2/Q9) $0.\overline{21} =$

(b) (10.2/Q10) 0.1244444... =

3. (10.2/Q13) In order to establish a **perpetual** fund that pays a **flat** amount of \$1000 at the end of every year, you invest a certain certain lump sum in an account paying 5% annual interest, compounded **monthly**, and leave it there without adding anything to it. Answer the following questions:

(a) What is the **present value** of the first payment? _____.

(b) What is the **present value** of the second payment? _____.

(c) What is the **present value** of the third payment? _____.

(d) Write down a geometric series that gives the value of the lump sum you must invest to establish the **perpetual** fund. State below its first term and the common ratio.

First term = _____. Common ratio = _____.

(e) What is value of the lump sum you must invest?

4. Suppose you want to a establish a **fifteen** year fund that pays out a certain amount at the end of every year, starting with \$5,000 at end of the first year and increasing the amount by 2% every year. Assuming that your investment pays 6% annual interest, compounded **continuously**, answer the following questions:

(a) What is the amount of the first payment? _____.

(b) What is the amount of the second payment? _____.

(c) What is the amount of the third payment? _____.

(e) What is the **present value** of the first payment? _____.

(f) What is the **present value** of the second payment? _____.

(g) What is the **present value** of the third payment? _____.

(h) Write down a geometric series that gives the value of the lump sum you must invest to establish the **fifteen year** fund. State below its first term, the common ratio, and the last term.

First term = _____. Common ratio = _____. Last term = _____.

(i) What is value of the lump sum you must invest?

5. (10.2/Q17) Suppose a patient take 10 milligrams of a certain anti-inflammatory steroid at the beginning of each day. The drug is eliminated exponentially from the patient's system in such a way that a single 10 milligram dose will leave $10e^{-0.8t}$ milligrams remaining in the patient's system after t days.

(a) By the end of the third day, the patient has taken a total of three doses. How many milligrams remain in his system?

(b) How many milligrams remain in the patient's system at the end of n days? Express your answer in terms of a finite geometric series and find its sum. Your answer will involve n.

(c) Assuming the treatment is continued indefinitely, use an infinite geometric series to approximate the amount remaining in the patient's system after a very long period.