

Multiple Choice Questions (5 points each)

Show **all** your work on the page where the question appears.

1. Compute the distance between the points $(-3, 0, 6)$ and $(-1, -2, 4)$.

(a) $\sqrt{10}$ (b) $\sqrt{11}$ (c) $\sqrt{12}$ (d) $\sqrt{13}$ (e) $\sqrt{14}$

2. Find

$$\lim_{h \rightarrow 0} \frac{\sqrt{x^2 + (y+h)^2} - \sqrt{x^2 + y^2}}{h}$$

(Hint: Think partial derivative.)

(a) $\frac{x}{\sqrt{x^2 + y^2}}$ (b) $\frac{y}{\sqrt{x^2 + y^2}}$ (c) $y\sqrt{x^2 + y^2}$
(d) $\frac{x}{2\sqrt{x^2 + y^2}}$ (e) $\frac{y}{2\sqrt{x^2 + y^2}}$

3. Let $f(x, y)$ be a function with

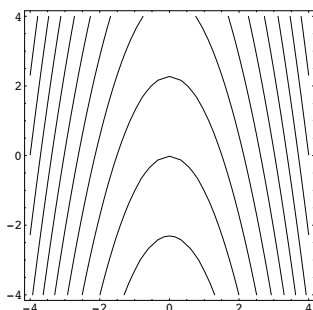
$$f(1, 0) = 2, \quad \frac{\partial f}{\partial x}(1, 0) = -1, \quad \frac{\partial f}{\partial y}(1, 0) = 3.$$

Find the equation of the tangent plane to the graph of $f(x, y)$ at the point $(1, 0)$.

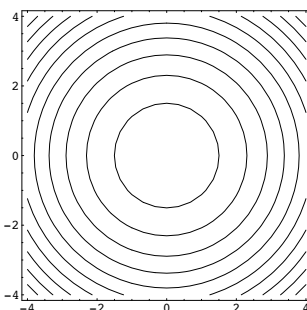
(a) $z = -x + 3y + 3$ (b) $z = -3x + y + 3$ (c) $z = x + 3y - 3$
(d) $z = x - 3y + 3$ (e) $z = 2$

4. Which of the following pictures shows level curves of the function

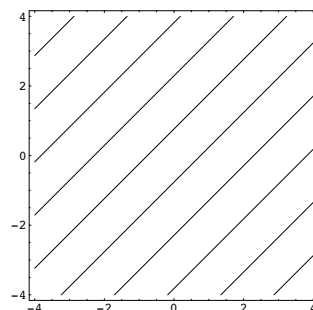
$$z = f(x, y) = y - x^2 + 2x?$$



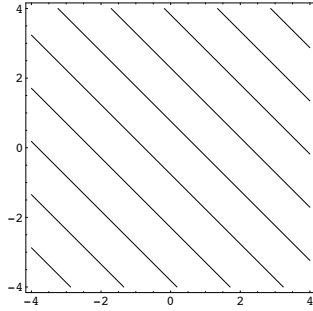
(a)



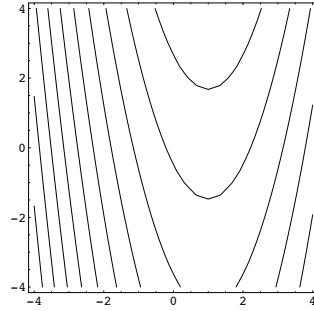
(b)



(c)



(d)



(e)

5. Let $f(x, y)$ be a function of two variables, and

$$f(3, 2) = 1, \quad \frac{\partial f}{\partial x}(3, 2) = 1, \quad \frac{\partial f}{\partial y}(3, 2) = -2.$$

Use linear approximation to estimate $f(3.1, 1.9)$.

- (a) 1 (b) 1.1 (c) 1.2 (d) 1.3 (e) 1.4

6. Find all critical points of the function $f(x, y) = y^3 + 3x^2 - 6xy - 9y - 2$.

- (a) $(-1, -2)$ and $(3, 3)$ (b) $(4, 4)$ and $(-2, -2)$ (c) $(-1, 3)$ and $(3, -1)$
 (d) $(-2, -1)$ and $(2, 3)$ (e) $(-1, -1)$ and $(3, 3)$

7. For the function $f(x, y) = xe^{y^3}$, calculate $\frac{\partial^2 f}{\partial x \partial y}$.

- (a) $3y^2 e^{y^3}$ (b) xye^{y^3} (c) $2ye^{y^3}$
 (d) $2ye^{y^2}$ (e) $3xye^{y^3}$

8. Let $f(x, y)$ be a function with a critical point at $(1, -1)$ and the second order partial derivatives of f at this point given by:

$$\frac{\partial^2 f}{\partial x^2}(1, -1) = -3, \quad \frac{\partial^2 f}{\partial y^2}(1, -1) = -6, \quad \frac{\partial^2 f}{\partial x \partial y}(1, -1) = 4.$$

Use the second derivative test to determine the nature of this critical point.

- (a) $f(x, y)$ has a local maximum at $(1, -1)$,
 (b) $f(x, y)$ has a local minimum at $(1, -1)$,
 (c) $f(x, y)$ has a saddle point at $(1, -1)$,
 (d) The test is inconclusive.
 (e) Such a function cannot exist.

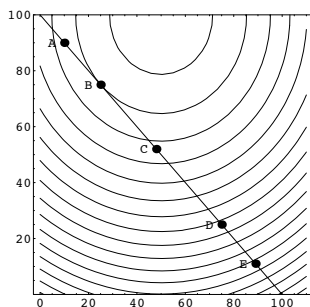
9. If you wanted to find the least-squares line of the form $y = ax + b$ for the given data

$$(2, 0), (-1, 1), (1, 2),$$

which of the following error functions would you need to minimize in order to determine the values of a and b ?

- (a) $E(a, b) = (a + 2b)^2 + (a - b + 1)^2 + (a + b - 3)^2$
 (b) $E(a, b) = (2a + b)^2 + (-a - b + 1)^2 + (a + b - 2)^2$
 (c) $E(a, b) = (2a + b)^2 + (-a + b - 1)^2 + (a + b - 2)^2$
 (d) $E(a, b) = (a + 2b)^2 + (-a - b + 1)^2 + (a + b - 2)^2$
 (e) $E(a, b) = (a - b + 3)^2 + (2b + 1)^2$

10. The graph below shows the line $x + y = 100$ and some level curve of $f(x, y) = 10x + 20y - 0.1(x^2 + y^2)$. The function has a maximum subject to constraint $x + y = 100$. At which point does it occur?



- (a) A (b) B (c) C (d) D (e) E

Partial Credit Questions

Show all work and put your final answer in the space provided. No credit will be given for a correct answer without showing how it was obtained. You will receive no credit if the answer is not in the space provided and no partial credit for a wrong answer if you do not show your work

11. Find the equation of the plane through the following three points

$$(1, 3, 2), \quad (3, -1, 6), \quad (5, 0, 0).$$

12. (a) Let $f(x, y) = x^3y + 12x^2 - 8y$. Find the critical points of f .
 (b) Let $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2$. The critical points of $f(x, y)$ are $(1, 1)$, $(1, -1)$, $(0, 0)$, and $(2, 0)$. Use the second derivative test, if possible, to determine the nature of each critical point. If the test is inconclusive, say so.

13. (a) Let $f(x, y) = \ln(x - 3y)$.
Find the linear approximation for $f(x, y)$ at the point $(7, 2)$.
- (b) When the company produces 150 units of X and 200 units of Y , its profit is \$40,000. The marginal profit with respect to X is \$100, and the marginal profit with respect to Y is \$150. Use linear approximation to estimate the company's profit when it produces 160 units of X and 190 units of Y .

14. A pharmaceutical company estimates that the demand function for one of its drugs in the U.S. is

$$p_1 = 25 - 0.2x,$$

while the demand function for the same drug in the Canadian market is

$$p_2 = 10 - 0.05y,$$

where x and y are in thousands of units and p_1 and p_2 are in dollars per unit. Its cost function is

$$C(x, y) = 15 + 0.2(x + y), \text{ in thousands of dollars.}$$

- (a) Write down a profit function as a function of x and y .
- (b) Find the quantities x and y that maximize the company's profit from the sale of this drug.
- (c) What are the corresponding prices?
15. A company's output is given by the Cobb-Douglas production function $P(K, L) = 100K^{1/3}L^{2/3}$, where K denotes capital and L denotes labor. Suppose the company's total budget for capital and labor is \$120,000, and each unit of capital costs \$20 and each unit of labor costs \$10.
- (a) Model this problem as a constrained optimization problem.
- (b) Find the combination of capital and labor that gives the maximum output by using the **Lagrange Multiplier Method**.