Chapter 10 Miscellaneous Problems

1. Suppose a smooth function $f(x)$ is such that $\quad f(0) = 0, \quad f'(0) = -2, \quad f''(0) = 1, \quad f'''(0) = 24$

(a) Write down the 3rd-degree Taylor polynomial of $f(x)$ about 0.

(b) Write down the 2nd-degree Taylor polynomial of $g(x) = e^{f(x)}$ about 0.

(c) Write down the 3rd-degree Taylor polynomial of $h(x) = f(x-1)$ about 1.

(d) Write down the 2nd-degree Taylor polynomial of $k(x) = x \cdot f(x)$ about 0.

2a. Write down the 5th-degree Taylor polynomial of $f(x) = e^x$ about 0.

2b. Estimate e with your result in Part (a). (Hint: Check your work against the calculator)

3. The 4th-degree Taylor polynomial of $f(x)$ about 2 is given by

$$P_4(x) = -3 + 2(x-2)^2 - 4(x-2)^3 + (x-2)^4.$$

(a) Write down the value of $f'(2)$. What can you say about the graph of $f(x)$ at $x = 2$?

(b) Write down the value of $f''(2)$. What can you say about the graph of $f(x)$ at $x = 2$? (Hint: $f''(2) = 4 > 0$)

(c) Write down the values of $f(2)$, $f'''(2)$, and $f^{(4)}(2)$.

4. Let $f(x)$ and $g(x)$ be two functions such that

$$f(2) = 0, \qquad f'(2) = 5, \qquad f''(2) = -6, \qquad f'''(2) = 18$$

$$g(2) = 1, \qquad g'(2) = -1, \qquad g''(2) = 2, \qquad g'''(2) = 12$$

(a) Find the 3rd-degree Taylor polynomial of $h(x) = f(x+2)$ about 0.

(b) Find the 2nd-degree Taylor polynomial of $k(x) = e^{f(x)}$ about 2.

(c) Find the 2nd-degree Taylor polynomial of $p(x) = f(x) \cdot g(x)$ about 2.

5. The 4th-degree polynomial of the cost function $C(x)$ (in millions of dollars) of a certain company about 1 is given by $\quad 3 + 2(x-1) + 5(x-1)^3 + 2(x-1)^4 \quad$ where x is the quantity of a product sold in millions of units.

(a) What is the marginal cost at $x = 1$?

(b) Find the equation of the tangent line to the graph of the cost function at $x = 1$. Give your answer in the form $y = mx + b$.

(c) Write down the values of $C(1)$, $C'(1)$, $C''(1)$, $C'''(1)$, and $C^{(4)}(1)$.

6. The 4th-degree polynomial of the function $f(x)$ about -2 is given by

$$P_4(x) = -1 + 2(x+2) - 3(x+2)^2 + 4(x+2)^4.$$

(a) What is the slope of $f(x)$ at $x = -2$? Find the equation of the tangent line to the graph of $f(x)$ at $x = -2$. Give your answer in the form $y = mx + b$.

(b) Write down the values of $f(-2)$, $f''(-2)$, $f'''(-2)$, and $f^{(4)}(-2)$.

7. Let $f(x) = \ln(3 - x)$ for $x < 3$.

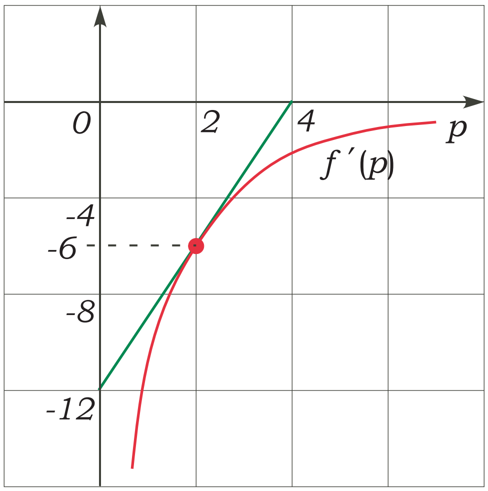
(a) Write down the 3rd-degree Taylor polynomial of $f(x)$ about 2.

(b) Use your result in (a) to estimate the value of $\ln(1.2)$.

(c) Find an error bound for the estimate you found in (b).

8. Demand function of a certain product is given by $q = f(p)$ where q number of items in hundreds and p is the price in thousands of dollars per unit. If the demand is 400 when each unit of the product is priced at two thousand dollars, and the graph of the **derivative** $f'(p)$ of $f(p)$ is as shown below, find the 2nd-degree Taylor polynomial of the **revenue** $R(p)$ function about $p = 2$ (thousand dollars).

(Hint: $R = p \cdot q$. Write down also the values of f and its derivatives at $p = 2$)



9. Consider the Taylor series for $\ln x$ about 1:

$$\ln x = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{(x-1)^k}{k} = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots \quad \text{for} \quad 0 < x \leq 2$$

(a) Estimate $\ln(1.4)$ with the 4th-degree Taylor Polynomial of $\ln x$ about 1.

(Hint: Check your work against the calculator)

(b) Write the error (remainder term) for the estimate in (a) as an infinite sum. Give at least the first three terms of the infinite sum.

(c) Write down the Taylor series for $\ln(x+2)$ about -1, stating the values of x for which the series is convergent.
(Hint: Replace x by $(x + 2)$ in the given series)

10. The amount of capital held by an economy at time t (in years) is given by $C(t)$. Suppose the labor (number of workers in units of millions) at time t is given by the function $L(t) = e^t$.

(a) Write down the capital stock $k(t)$ in terms of $C(t)$. Recall that capital stock is capital per unit of workers.

(b) Find the 2nd-degree Taylor polynomial of $k(t)$ about 0 if $C(0) = 3$, $C'(0) = -1$, $C''(0) = 1$.

(Ans: $P_2(t) = 3 - 4t + 3t^2$)

11. Given that the Taylor series for some common functions about 0, and their values of x for which they converges are given below:

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots \quad = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^k}{k} \qquad \text{for} \quad -1 < x \leq 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad = \sum_{k=0}^{\infty} \frac{x^k}{k!} \qquad \text{for} \quad -\infty < x < \infty$$

(a) Using the appropriate series above, write down the Taylor series for $\ln(x-2)$ about 3 stating the values of x for which the series is convergent.

(b) Estimate $\ln(1.3)$ with the 5th-degree Taylor Polynomial of $\ln(1+x)$ about 0.

(c) Estimate e^2 with the 5th-degree Taylor Polynomial of e^x about 0.