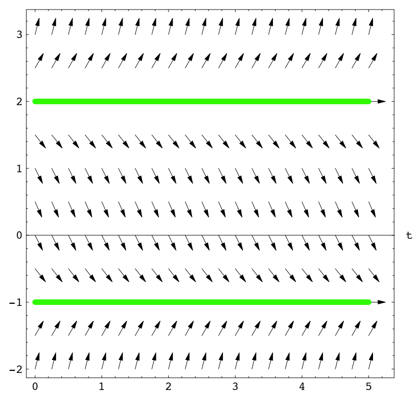
Part I: Multiple choice questions (5 points each)

1. Some rangers estimate that in their park the deer population $p(t)$, in thousands, is governed by the logistic differential equation $\frac{dp}{dt} = 0.1p(1-p)$ and that the current size of the population is 0.4, that is $p(0) = 0.4$. Using Euler's Method with $\Delta t = 5$, they predict that the deer population (in thousands) 10 years from now is:

   (a) 0.2400 (b) 0.5225 (c) 0.6448 (d) 0.4484 (e) 0.7664

2. Consider the following direction field:



   Describe the equilibrium solutions to the appropriate differential equation $\frac{dy}{dt} = g(y)$.

   (a) There is an unstable equilibrium solution at $y = 2$ and a stable equilibrium solution at $y = -1$

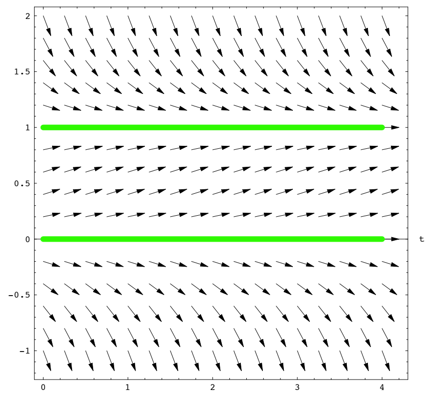
   (b) There is a stable equilibrium solution at $y = 2$ and an unstable equilibrium solution at $y = -1$

   (c) There are two stable equilibrium solutions at $y = 2$ and at $y = -1$

   (d) There are two unstable equilibrium solutions at $y = 2$ and at $y = -1$

   (e) The equation has no equilibrium solutions

3. Given the following direction field, find the matching differential equation.



(a) $y' = y(y-1)$

(b) $y' = y(1-y)$

(c) $y' = \dfrac{1}{y(1-y)}$

(d) $y' = 1 - e^{t(1-y)}$

(e) $y' = \dfrac{y}{y-1}$

4. A population of a certain kind of animal is modeled by:

$$\frac{dp}{dt} = -\ln\left(\frac{p}{150}\right),$$

where $p(t)$ is in millions. If the current size of the population is about 40 million, its limiting value (in millions) as t goes on is:

(a) $\dfrac{1}{150}$ (b) 0 (c) $+\infty$ (d) 150 (e) $-\ln\left(\dfrac{1}{150}\right)$

5. Find the 3rd degree Taylor polynomial of $f(x) = e^{5x}$ at $a = 0$.

(a) $1 + 5x + \frac{25}{2}x^2 + \frac{125}{6}x^3$

(b) $1 + x + x^2 + x^3$

(c) $1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$

(d) $1 + (x-1) + \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3$

(e) $1 + 5x + 25x^2 + 125x^3$

6. Find the infinite sum:

$$\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k = \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \ldots$$

(a) 4 (b) $\dfrac{1}{4}$ (c) $\dfrac{111}{64}$ (d) 3 (e) The series is divergent.

7. Find the Taylor series expansion of $f(x) = \dfrac{4}{1+x}$ about $x = 0$.

(a) $\sum_{k=0}^{\infty} \frac{(-4)^k}{k!} x^k = 1 - 4x + 8x^2 - \frac{32}{3}x^3 + \cdots$

(b) $\sum_{k=0}^{\infty} 4x^k = 4 + 4x + 4x^2 + 4x^3 + \cdots$

(c) $\sum_{k=0}^{\infty} 4(-1)^k x^k = 4 - 4x + 4x^2 - 4x^3 + \cdots$

(d) $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^k = -1 + x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \cdots$

(e) None of the others

8. Suppose a patient is injected with a drug once a day. Assume the amount of a single injection remaining after t days is given by $Q(t) = 50e^{-0.2t}$. How much of the drug is remaining in the patient's system after a very long time, assuming that measurements are being done just before each injection? (**Hint:** use geometric series.)

(a) $\dfrac{50}{1+e^{-0.2}}$ (b) $\dfrac{50\,e^{-0.2}}{1-e^{-0.2}}$ (c) $\dfrac{50}{1-e^{0.2}}$ (d) 0 (e) None of the others

9. Suppose you roll a pair of fair dice. What is the probability that the sum of the top faces is five?

(a) $\dfrac{1}{12}$ (b) $\dfrac{6}{11}$ (c) $\dfrac{1}{4}$ (d) 6 (e) $\dfrac{1}{9}$

10. Suppose E and F are events satisfying:

$$P(E) = 0.4, \qquad P(E \cup F) = 0.7, \qquad P(E \cap F) = 0.2.$$

What is $P(F)$?

(a) 0.5 (b) 1 (c) 0.2 (d) 0.4 (e) cannot be determined

Part II: Partial credit questions (10 points each)

Show all of your work!!!

11. Consider the Solow differential equation:

$$\frac{dk}{dt} = 0.3\sqrt{k} - 0.15k,$$

with initial condition:

$$k(0) = 2.25.$$

Use Euler's method with $n = 3$ to approximate $k(12)$.

12. Consider the differential equation:

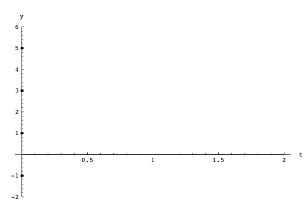
$$\frac{dy}{dt} = y(4 - y) = 4y - y^2.$$

(a) Find the equilibrium solutions, draw their graphs in the ty-coordinate system provided below, and decide the stability of these solutions.

(b) Use the graphical method to sketch the graph of the solution to the given initial value problem:

$$y(0) = -1, \qquad y(0) = 1, \qquad y(0) = 3, \qquad y(0) = 5.$$

In all cases provide explanations for the shape of the graph. (Make sure to sketch the graph of the inflection line in the ty-coordinate system and show concavity clearly.)



13. (a) Estimate the integral $\int_0^1 \ln(1 + x)\, dx$ by using the 2^{nd} degree Taylor polynomial of $f(x) = \ln(1 + x)$ about $a = 0$.

(b) Suppose $y(t)$ is the solution to the initial value problem:

$$y' = 3t - y, \qquad y(0) = 5.$$

Estimate $y(0.2)$ by using its 2^{nd} degree Taylor polynomial.

14. You want to establish a perpetual fund that pays out a certain amount at the end of every year according to the following plan:

- the fund pays $\$10,000$ at the end of the first year;
- the amount paid is increased by 4% every subsequent year.

How much must you endow the fund with if your investment pays 10% annual interest, compounded continuously?

15. The probability table of an experiment with outcomes s_1, s_2, s_3, s_4 is given below:

outcome	s_1	s_2	s_3	s_4
probability	0.2	0.5	0.25	0.05

Let $E = \{s_2, s_3\}$ and $F = \{s_3, s_4\}$.

(a) Find $P(E)$.

(b) Find $P(F)$.

(c) Find $P(E \cap F)$.

(d) Find $P(E \cup F)$.