Math 108 Exam 1 - Solutions - Spring 2005

1. P(40)-P(30)=$\int_{30}^{40}(-0.03x^2-3x+525)dx = (-0.01x^3-\frac{3}{2}x^2+525)|_{30}^{40}$
$= [-0.01(40)^3-\frac{3}{2}(40)^2+525\cdot 40) - [0.01\cdot(30)^2-\frac{3}{2}(30)^2+525\cdot 30] = 3,830.$
2. Letting $u = x^3-3x^2+1$ gives $du = (3x^2-6x)dx$, or $(x^2-2x)dx = \frac{1}{3}du$ and the given integral takes the form $\int e^u\frac{1}{3}du = \frac{1}{2}\int e^u du.$
3. Letting $u = t^2$ and $dv = e^{0.1t}dt$ gives $du = (t^2)^1 dt = 2tdt$, and $v = \int e^{-0.1t}dt = 10e^{0.1t}$. Thus applying integration by parts we obtain
$\int t^2e^{0.1t}dt = t^2\cdot 10e^0.1t\int 10e^{0.1t}2tdz = 10t^2e^{0.1t}20\int te^{0.1t}dt.$
4. $\int_0^3 f(x)dx \approx (6+5+4+2.5-1-3)\cdot(0.5) = 6.75$
5. The average over [0.6] is $\frac{1}{6-0}\int_0^6 e^{0.07t}dt =$
$= \frac{1}{6}\frac{1}{0.07}e^{0.07}|_0^6 =?$
6. PV=$\int_0^\infty 30,000e^{-0.06t}dt = \lim_{T\to}\int_0^T 30,000e^{-0.06t}dt$
$= \lim_{T\to\infty}\frac{30,000}{-0.06}e^{-0.06t}|_0^T = -500,000\lim_{T\to\infty}(e^{-0.06T}-1)$
$= -500,000(0-1) = 500,000.$
7. $\int_0^2 f(x)dx \approx (7+6.4+5+3)(0.5) = 10.7$
8. Separating variables gives $\frac{dy}{y-1} = -2xdx$ and integrating gives
$\int\frac{dy}{y-1} = \int -2xdx$, or $ln|y-1| = -x^2+c$,
or $|y-1| = e^{-x^2+c}$, or $y-1 = \pm e^c e^{-x^2}$.
Replacing $\pm e^c$ with $\mathbb{C}$ we obtain $y-1 = \mathbb{C}e^{-x^2}$
or $y(x) = 1+\mathbb{C}e^{-x^2}$. Since $3 = y(0) = 1+c\cdot e^0$
we obtain $3 = 1+\mathbb{C}$ or $\mathbb{C} = 2$. Thus $y(x) = 1+2e^{-x^2}$.
9. $\frac{dp}{dt} = 0.014p(1-\frac{p}{12})$.
10. 430 must be equal to the present value of 215 plus the present value of the income stream of rate $\mathbb{S}$ billion per year for 4 years. Thus, we must have
$430 = 215e^{-0.05\cdot 4}+\int_0^4\mathbb{S}e^{-0.05t}dt$, or
$430 = 215e^{-0.2}+\mathbb{S}\frac{e^{-0.05}}{-0.05}|_0^4$, or
$430 = 215e^{-0.2}+\frac{\mathbb{S}}{0.05}(1-e^{-0.2})$
Solving for $\mathbb{S}$ gives $\mathbb{S} = \frac{0.05(430-215e^{-0.2})}{1-e^{-0.2}}$.
11. (a) The area under the demand curve over [0,5] is approximately equal to 25+19+14+20+7=85. Therefore
$CS = \int_0^5 D(q)dq - 5\cdot 7 \approx 85-35 = 50.$
(b) Solving $D(q) = S(q)$, or $\frac{56}{q+2} = q+3$, or $q^2+5q+6 = 56$, or $q^2+5q-50 = 0$, or $(q+10)(q-5) = 0$. This gives $q_e = 5$, since $q = -10$ is not acceptable. Thus $p_e = 5+3 = 8$. The producer surplus is the area of the right triangle with vertices at (0,3),(0,8) and (5,8), which is $\frac{1}{2}(8-3)\cdot 5 = 12.5$.
12. (a) $\frac{1}{(x+1)(x-2)} = \frac{A}{x+1}+\frac{B}{x-2}$ gives $1 = A(x-2)+B(x+1)$.
Letting $x = -1$ gives $1 = A(-3)$ or $A = -\frac{1}{3}$.

Letting $x = 2$ gives $1 = 3B$ or $B = \frac{1}{3}$.

(b) $\int \frac{1}{p(10-p)} dp = \frac{1}{10} \int \frac{1}{p} dp - \frac{1}{10} \int \frac{1}{p-10} dp$
$= \frac{1}{10} \ell n|p| - \frac{1}{10} \ell n|p - 10| + c = \frac{1}{10} \ell n|\frac{p}{p-10}| + c.$

13. (a) $\frac{dp}{dt} = 0.14p, p(0) = 200.$

(b) $\frac{dM}{dt} = 0.05M - 18,000, p(0) = 200,000$

14. FV$=\int_0^{20} 10,000e^{0.1t} e^{0.05(20-\alpha)} dt$
$= 10,000e \int_0^{20} e^{0.05t} dt = 10,000e \frac{e^{0.05t}}{0.05} |_0^{20}$
$= 200,000e(e - 1) =?$
We will choose (a) since is bigger.

15. (a) $FV_a = \int_0^{10} 0.5e^{0.03(10-t)} dt = 5.830$

(b) $FV_b = 1.5e^{0.03*10} + \int_0^{10} 0.75e^{0.03(10-t)} dt = 10.77$
Surplus $= FV_b - FV_a = 10.77 - 5.830 = 4.94$ trillions.