

Part I: Multiple choice questions (5 points each)

Show **all** your work on the page on which the question appears.

1. A company producing 30 units per day of a software package estimates that its marginal profit (in dollars per unit) is given by

$$P'(x) = -0.03x^2 - 3x + 525.$$

What will the change in profit be if the company increases production to 40 units per day?

- (a) -0.60 (b) -51 (c) $\$357$ (d) $\$3,830$ (e) $\$17,960$

2. If in the indefinite integral $\int e^{x^3-3x^2+1} \cdot (x^2 - 2x) dx$ you make the substitution $u = x^3 - 3x^2 + 1$ then you will obtain the integral:

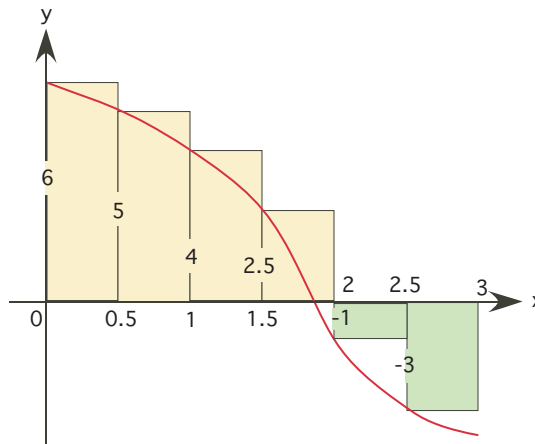
- (a) $\frac{1}{3} \int e^u \cdot du$ (b) $\int e^u \cdot du$ (c) $3 \int e^u \cdot du$ (d) $\frac{1}{3} \int e^{u^3} \cdot du$ (e) $3 \int e^{u+1} \cdot du$

3. If in the definite integral $\int t^2 \cdot e^{0.1t} dt$ you apply the integration by parts formula $\int u \cdot dv = uv - \int v \cdot du$ with $dv = e^{0.1t} dt$ then you will find that it is equal to:

- (a) $t^2 e^{0.1t} - \int t \cdot e^{0.1t} dt$ (b) $10t^2 e^{0.1t} - \int t \cdot e^{0.1t} dt$ (c) $t^2 e^{0.1t} - 20 \int t \cdot e^{0.1t} dt$
 (d) $10t^2 e^{0.1t} - 20 \int t \cdot e^{0.1t} dt$ (e) $10t^2 e^{0.1t} - 20 \int e^{0.1t} dt$

4. For the function $f(x)$ whose graph is displayed in the figure on the right, estimate $\int_0^3 f(x) dx$ by using the Riemann sum corresponding to $\Delta x = 0.5$ and the left-hand endpoints.

- (a) -6.75 (b) 10.75
 (c) 6.75 (d) -21.75
 (e) None of the above.



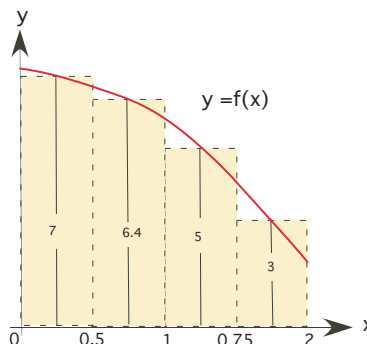
5. Suppose you deposit $\$5,000$ into an account paying an annual rate of interest of 7%, compounded continuously. No further deposits or withdrawals are made. What is the **average** amount of money in the account during the first 6 years?

- (a) $\$1,268.30$ (b) $\$5,350.64$ (c) $\$7,609.81$ (d) $\$6,420.76$ (e) $\$6,213.83$

6. A business is expected to produce a **perpetual** income flowing continuously at a rate of \$30,000 per year. The income from the business will be invested at an annual rate of interest of 6%, compounded continuously. What is the present value of this income stream?

- (a) \$500,000 (b) \$16,667 (c) \$180,000 (d) \$1,800
 (e) Present value cannot be calculated in this case.

7. For the function $f(x)$ whose graph is displayed in the figure on the right, estimate $\int_0^2 f(x)dx$ by using the the midpoint rule.



- (a) 21.4 (b) 10.7
 (c) -21.4 (d) -10.7
 (e) None of the above.

8. Solve the differential equation $\frac{dy}{dx} = -2x(y - 1)$ with the initial condition $y(0) = 3$.

- (a) $y = 5 - 2e^{-x^2}$ (b) $y = 1 + 2e^{-x^2}$ (c) $y = 2 + e^{-x^2}$ (d) $y = 4 - e^{-x^2}$ (e) $y = 1 + 2e^{x^2}$

9. Write a logistic differential equation that models the world population if you assume that its intrinsic growth rate is 1.4% and that Earth's carrying capacity is 12 billion.

- (a) $\frac{dp}{dt} = p(1 - \frac{p}{12})$ (b) $\frac{dp}{dt} = 0.014p(1 - 12p)$ (c) $\frac{dp}{dt} = 0.014p(1 - \frac{p}{12})$
 (d) $\frac{dp}{dt} = 0.014p(1 + \frac{p}{12})$ (e) None of the above

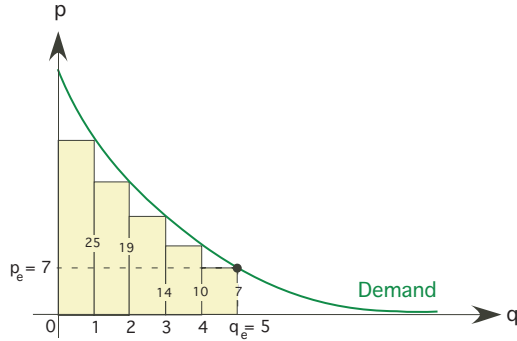
10. The U.S. deficit is now about \$430 billion. The President said that he wants to cut it in half during the next four years. Assume that he plans to do it by making steady payments at the rate of S billion dollars per year. If the only way that the deficit grows is by earning interest at the annual rate of 5% compounded continuously, find the value of S . (Hint: 430 is equal to the PV of 215 plus the PV of an income stream! Also, you can solve it by using FV or differential equations.)

- (a) $S = \frac{0.05(430 - 215e^{-0.2})}{1 - e^{-0.2}}$ (b) $S = \frac{430 - 215e^{-0.2}}{1 - e^{-0.2}}$ (c) $S = \frac{0.05(430 - 215e^{-0.2})}{1 + e^{-0.2}}$
 (d) $S = \frac{0.05(430 + 215e^{-0.2})}{1 - e^{-0.2}}$ (e) None of the above

Part II: Partial credit questions (10 points each)

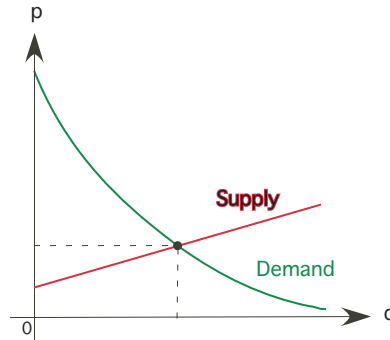
Show all work and put your final answer in the space provided. No credit will be given for a correct answer without showing how it was obtained. You will receive no credit if the answer is not in the space provided and no **partial credit** for a wrong answer if you do not **show your work**.

11. (a) For the demand curve displayed in the figure on the right, estimate the consumer surplus by using Riemann Sums corresponding to $\Delta q = 1$ and right-hand endpoints. Observe that the equilibrium quantity is 5 and the equilibrium price is 7.



Answer: $CS \stackrel{?}{=}$

- (b) The demand curve of a certain item is $p = D(q) = \frac{56}{q+2}$ and its supply curve is given by $p = S(q) = q + 3$. Find the equilibrium price and the equilibrium quantity and compute the producer surplus. Finally, in the figure on the right, indicate the region (by shading it) whose area is the producer surplus.



Answer: $q_e \stackrel{?}{=}$ $p_e \stackrel{?}{=}$ $PS \stackrel{?}{=}$

12. (a) Write $\frac{1}{(x+1)(x-2)}$ in the form $\frac{A}{x+1} + \frac{B}{x-2}$.

(b) Compute $\int \frac{1}{p(10-p)} dp$ (Hint: $\frac{1}{p(10-p)} = \frac{1/10}{p} - \frac{1/10}{p-10}$)

13. (a) At any time t , a population p grows at a rate proportional to its current size, with a constant of proportionality equal to 0.14. Also, its size now is equal to 200. Use a differential equation and an initial condition to model this population. (Do NOT solve it!)

- (b) You have just borrowed \$200,000 to buy a house at an annual interest rate of 5% compounded continuously. Suppose you can make payments at the steady rate of \$18,000 per year. Use a differential equation and an initial condition to model the amount $M(t)$ you will owe after t years. (Do NOT solve it!)

14. Your company offers you the following two options:

(a) For the next 20 years it deposits money continuously into an account A at a rate of $10,000e^{0.1t}$ dollars per year.

(b) At the beginning it deposits \$325,000 into an account B and nothing more during the next 20 years.

If both accounts yield annual interest of 5%, compounded continuously, which option will you choose? Explain your answer. (Hint: Compute PV of the income stream in (a).)

15. (a) During 2004 about 48 million Americans—not just retired workers but also the disabled and the spouses and children of deceased workers—drew about \$0.5 trillion per year in benefits from Social Security¹. Assume that for the next 10 years Social Security continues to pay benefits at this rate (\$0.5 trillion) per year steadily.

(b) On the other hand, at the moment, the Social Security has a surplus (Trust-fund) of about \$1.5 trillion, and an income stream of about \$0.75 trillion per year. Assume, again, that for the next 10 years Social Security continues to receive this income at the same rate (\$0.75 trillion) per year steadily.

Find the the Social Security surplus after 10 years, if during this period the prevailing annual interest rate is 3% compounded continuously. (Hint: Compute the difference of the two FV's.)

¹Source: TIME: January 24, 2005