Department of Mathematics
University of Notre Dame
Math 10-260 – Bus. Calc. 2
Spring 2008

Name:____________________________

Instructor:__________________________

Exam I

February 7, 2008

This exam is in 2 parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. May the force be with you.

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You must record here your answers to the multiple choice problems.
Place an × through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
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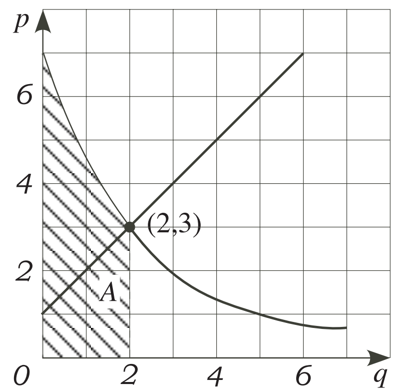
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	________
11.	________
12.	________
13.	________
14.	________
15.	________
Tot.	________

Multiple Choice

1. (5 pts.) The demand and supply curves of a soft drink is given below. If the area of the shaded region A is 10, what is the **consumer surplus** and **producer surplus** ?



(a) Consumer surplus = 1 and Producer surplus = 2.

(b) Consumer surplus = 2 and Producer surplus = 4.

(c) Consumer surplus = 4 and Producer surplus = 2.

(d) Consumer surplus = 4 and Producer surplus = 1.

(e) Consumer surplus = 1 and Producer surplus = 4.

2. (5 pts.) The money market account of a businessman earns interest at an annual rate r compounded **monthly**. If the balance of the account **doubles** in 10 years, what is the value of r?

(a) $r = 12\left(2^{(1/120)} - 1\right)$ (b) $r = \frac{1}{10}\ln 2$ (c) $r = 2^{(1/120)} - 1$

(d) Cannot be determined. (e) $r = \ln 2 - 10$

3. (5 pts.) The graph of the value $V(t)$ of a stock from the beginning of January to the beginning of May last year is given below. Using a Riemann sum with $\Delta t = 1$ and right endpoints, estimate the **average value** of the stock over the time period from the beginning of January ($t = 0$) to the beginning of May ($t = 4$) last year.

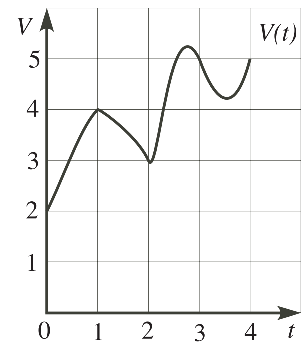
(a) 17

(b) 14

(c) 7/2

(d) 3/4

(e) 17/4



4. (5 pts.) The value of a certain make of computer changes at a rate of $r(t) = 3t^2 - 154$ dollars per month for $0 \leq t \leq 24$. What is the total change in the value of the computer over the **first 12 months**?

(a) \$30

(b) $-$\$120

(c) $-$\$72

(d) \$72

(e) \$120

5. (5 pts.) When evaluating the integral $\int_0^1 x\sqrt{2x+1}\,dx$ using the **method of substitution** with $u = 2x+1$, you get the expression:

(a) $\frac{1}{4}\int_0^1 (u-1)\sqrt{u}\,du$

(b) $\frac{1}{2}\int_0^1 (u-1)\sqrt{u}\,du$

(c) $\frac{1}{2}\int_1^3 (u-1)\sqrt{u}\,du$

(d) $\frac{1}{4}\int_1^3 (u-1)\sqrt{u}\,du$

(e) $\frac{(5)^{3/2}}{3} - \frac{1}{3}\int_0^1 (2u+1)^{3/2}\,du$

6. (5 pts.) Suppose the population of a region of the world grows according to the logistic differential equation

$$\frac{dp}{dt} = 0.05p - 0.002p^2$$

What is the carrying capacity K of the population?

(a) $K = 0.02$

(b) $K = 20$

(c) $K = 25$

(d) $K = 500$

(e) $K = 0.04$

7. (5 pts.) A young executive deposits continuously at the rate $(5t+3)$ thousand dollars per year into an account paying 6% compounded continuously. Which one of the following expressions give the balance of the account after 5 years, in thousands of dollars?

(a) $\int_0^5 (5t+3)e^{0.06(5-t)}\,dt$

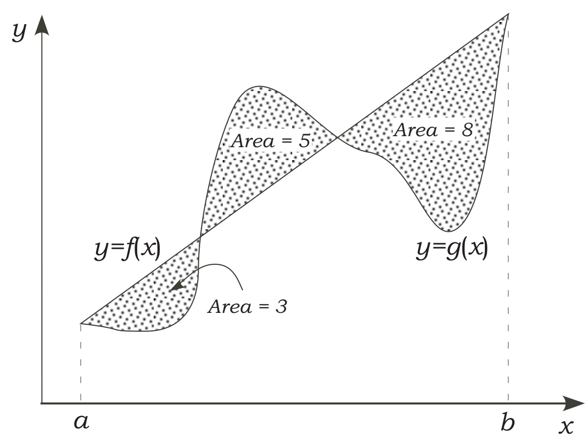
(b) $\frac{5t+3}{0.06}(1-e^{-0.3})$

(c) $\int_0^5 (5t+3)e^{-0.06t}\,dt$

(d) $\frac{5t+3}{0.06}(e^{0.3}-1)$

(e) $2e^{0.3}+7e^{0.24}+12e^{0.18}+17e^{0.12}+22e^{0.06}$

8. (5 pts.) The areas between the functions $f(x)$ and $g(x)$ for $a \le x \le b$ are given below.



If $y = f(x)$ is the linear function, find the value of $\int_a^b [f(x) - g(x)]\,dx$.

(a) -6 (b) 0 (c) 16

(d) 6 (e) -16

9. (5 pts.) An ND freshman decides to open a savings account. Starting on March 1st 2008, and for the **next three** years (2009 through 2011) on that date, she deposits \$500 into an account paying 6% interest compounded continuously. She plans to close the account and withdraw all the money on March 1st 2012. What is the **future value** of the **second** deposit at the time of withdrawal?

(a) $500e^{0.24}$ (b) $500e^{-0.06}$ (c) $500e^{0.18}$

(d) $500e^{-0.18}$ (e) 500

10. (5 pts.) For the same scenario above, what is the balance of the savings account at withdrawal?

(a) $\int_0^4 500e^{0.06(4-t)}dt$

(b) $500e^{0.18} + 500e^{0.12} + 500e^{0.06} + 500$

(c) $500e^{0.24} + 500e^{0.18} + 500e^{0.12} + 500e^{0.06}$

(d) $\int_0^4 500e^{-0.06t}dt$

(e) $500 + 500e^{-0.06} + 500e^{-0.12} + 500e^{-0.18}$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) A colony of wasps is building an underground nest. They start by finding an **empty** hole and begin constructing cells at a rate of

$$\frac{dy}{dt} = e^{-0.5y}$$

in tens of cells per day. Find the number of cells y in terms of t. Assume that $y(0) = 0$

12. (10 pts.)
Part A. (5 pts.) **Without using a calculator**, work out the integral:

$$\int x \ln x \, dx$$

Part B. Dave takes up a three year car loan of \$10,000 at annual interest rate of 8% compounded continuously. Assuming that payments are made continuously at a constant rate of S dollars per year, answer the questions below.

B (i) (3 pts.) If $M(t)$ denotes the balance of the loan at time t $(0 \leq t \leq 3)$, write down but **do not solve** a differential equation satisfied by $M(t)$. Your answer should involve S.

B (ii) (2 pts.) Write down both conditions satisfied by $M(t)$ so that you could solve for **BOTH** $M(t)$ and S in B (i).

13. (10 pts.)

Part A. (5 pts.) Find the partial fraction decomposition for $\frac{22}{x^2 - 7x - 18}$.
(You do not need to integrate your answer.)

Part B. (5 pts.) **Without using a calculator**, find the equilibrium point for the demand and supply functions below:

$$D(q) = (q - 5)^2; \qquad S(q) = q^2 + q + 3$$

14. (10 pts.)
Part A. Without using a calculator, work out the integral:

$$\int x\sqrt{1-x^2}\,dx$$

Part B. Write down but **do not evaluate** an integral that gives the **present value** of a **perpetual** income stream flowing continuously at a rate of $e^{0.01t}$ thousand dollars per year and with interest compounded continuously at the rate of 5%.

15. (10 pts.)
Part A. Give the limit definition of the following improper integral:

$$\int_1^\infty \frac{4}{x^3}\,dx \stackrel{?}{=} \underline{\hspace{6cm}}$$

Part B. Without using a calculator, evaluate the improper integral below showing clearly how you use limits:

$$\int_1^\infty \frac{4}{x^3}\,dx$$

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MC.	______
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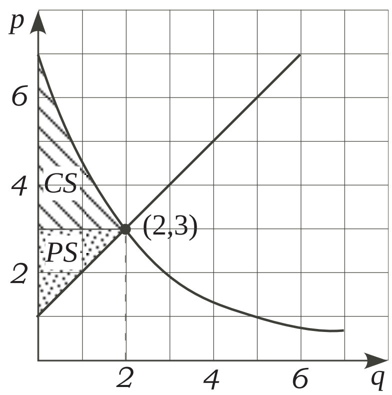
Tot.	______

Math 10260 Exam 1 (Bus. Calc. 2) Solution – Spring 2008

1.



Consumer supply $= 10 - 2(3) = 4$. Producer supply $= \frac{1}{2}(2)(2) = 2$.

2. Let P be the present value of the account. Then in 10 years, $2P = P\left(1+\frac{r}{12}\right)^{12\cdot 10} \Rightarrow 2 = \left(1+\frac{r}{12}\right)^{120} \Rightarrow$ $2^{\frac{1}{120}} = 1 + \frac{r}{12} \Rightarrow r = 12(2^{\frac{1}{120}} - 1)$.

3. Average of $V(t)$ for $0 \le t \le 4$ is $\frac{\int_0^4 V(t)dt}{4-0} \approx \frac{V(1)\Delta t + V(2)\Delta t + V(3)\Delta t + V(4)\Delta t}{4-0} = \frac{4+3+5+5}{4} = \frac{17}{4}$.

4. Let $V(t)$ be the value of the computer at time t. Then $V'(t) = r(t) = 3t^2 - 154$. By Fundamental Theorem of Calculus, $V(12) - V(0) = \int_0^{12} r(t)dt = \int_0^{12} 3t^2 - 154dt = t^3 - 154t|_0^{12} = 12^3 - 154 \cdot 12 = -120$.

5. Substituting $u = 2x+1$ we have $x = \frac{1}{2}(u-1)$, so $dx = \frac{1}{2}du$. If $x = 1$, then $u = 2\cdot 0 + 1 = 1$, and if $x = 1$, then $u = 3$.
   Therefore $\int_0^1 x\sqrt{2x+1}dx = \frac{1}{2}\int_1^3 \frac{1}{2}(u-1)\sqrt{u}du = \frac{1}{4}\int_1^3 (u-1)\sqrt{u}du.$

6. $\frac{dp}{dt} = 0.05p - 0.002p^2 \Rightarrow \frac{dp}{dt} = 0.05p(1 - \frac{0.002}{0.05}p) \Rightarrow K = \frac{0.05}{0.002} = 25.$

7. This is a continuous income stream problem asking for future value after 5 years. Therefore the required balance is $\int_0^5 (5t+3)e^{0.06(5-t)}\,dt$.

8. $\int_a^b [f(x) - g(x)]dx = 3 - 5 + 8 = 6$.

9. The second deposit is left on the account for three years (2009- 2012). Thus FV of the second deposit at the time of withdrawal $= 500e^{0.06(3)} = 500e^{0.18}$.

10. Deposits were made four times each a year apart then left in the account for one year.
    (i) FV of the initial deposit on March 1st 2012 $= 500e^{0.06(4)} = 500e^{0.24}$.
    (ii) FV of the second deposit on March 1st 2012 $= 500e^{0.06(3)} = 500e^{0.18}$.
    (iii) FV of the third deposit on March 1st 2012 $= 500e^{0.06(2)} = 500e^{0.12}$.
    (iv) FV of the fourth deposit on March 1st 2012 $= 500e^{0.06(1)} = 500e^{0.06}$.
    The balance of the savings account at withdrawal $= 500e^{0.24} + 500e^{0.18} + 500e^{0.12} + 500e^{0.06}$.

11. We have the initial value problem $\frac{dy}{dt} = e^{-0.5y}$ and $y(0) = 0$. Solve the differential equation $\frac{dy}{dt} = e^{-0.5y} \Rightarrow e^{0.5y}dy = dt \Rightarrow \int e^{0.5y}dy = \int dt \Rightarrow 2e^{0.5y} = t + C \Rightarrow 0.5y = \ln(0.5(t + C)) \Rightarrow y = 2\ln(0.5t + C')$, where $C' = 0.5C$.

    Finally, to evaluate C', substitute $y(0) = 0$, which gives $0 = 2\ln(0.5(0) + C') \Rightarrow 0 = 2\ln(C') \Rightarrow C' = 1$. Therefore $y = 2\ln(0.5t + 1)$.

12. A. $u = \ln x \Rightarrow du = \frac{1}{x}dx$; $dv = xdx \Rightarrow v = \frac{x^2}{2}$. Integration by parts formula gives:

    $\int x\ln x dx = \frac{x^2 \ln x}{2} - \int \frac{x^2}{2x}dx = \frac{x^2 \ln x}{2} - \frac{1}{2}\int xdx = \frac{x^2 \ln x}{2} - \frac{1}{4}x^2 + C.$

    B(i) $\frac{dM}{dt} = 0.08M - S.$

    B(ii) $M(0) = 10,000$, and $M(3) = 0$.

13. A. Since $x^2 - 7x - 18 = (x+2)(x-9)$, we have $\frac{22}{x^2 - 7x - 18} = \frac{A}{x+2} + \frac{B}{x-9}$. So $22 = A(x-9) + B(x+2)$. If $x = 9 \Rightarrow 22 = 11B \Rightarrow B = 2$, and if $x = -2 \Rightarrow 22 = -11A \Rightarrow A = -2$. Finally,

    $$\frac{22}{x^2 - 7x - 18} = -\frac{2}{x+2} + \frac{2}{x-9}.$$

    B. At equilibrium, $(q-5)^2 = q^2 + q + 3 \Rightarrow q^2 - 10q + 25 = q^2 + q + 3 \Rightarrow 11q = 22 \Rightarrow q = 2$. Hence $q_e = 2$ and $p_e = (2-5)^2 = 9$.

14. A. Let $u = 1 - x^2$, then $du = -2xdx \Rightarrow -\frac{1}{2}du = xdx$.

    $$\int x\sqrt{1 - x^2}dx = -\frac{1}{2}\int \sqrt{u}\, du = -\frac{1}{2} \cdot \frac{2}{3}u^{\frac{3}{2}} + C = -\frac{1}{3}(1 - x^2)^{\frac{3}{2}} + C.$$

    B. $\int_0^{\infty} e^{0.01t}e^{-0.05t}dt = \int_0^{\infty} e^{-0.04t}dt.$

15. A. $\int_1^{\infty} \frac{4}{x^3}dx = \lim_{T\to\infty} \int_1^T \frac{4}{x^3}dx.$

    B. $\int_1^{\infty} \frac{4}{x^3} = \lim_{T\to\infty} \int_1^T \frac{4}{x^3} = \lim_{T\to\infty} \left[\frac{-2}{x^2}\right]_1^T = \lim_{T\to\infty} \left(\frac{-2}{T^2} - \frac{-2}{1^2}\right) = -\frac{-2}{1^2} = 2$ since

    $$\lim_{T\to\infty} \frac{-1}{T^2} = 0.$$

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Exam II

March 13, 2008

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4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
15.	______
Tot.	______

Multiple Choice

1. (5 pts.) The daily low temperatures (in ^{o}C) for three days last week in South Bend is given in the table below:

Day (x)	1	3	4
Daily Low Temperature (y)	5	-2	6

Which of the functions below is the error function that needs to be minimized in order to determine the least squares line $y = ax + b$ for the data above?

(a) $E(a, b) = (a + b - 5) + (2a + b + 2) + (3a + b - 6)$

(b) $E(a, b) = (a + b + 5)^2 + (3a + b + 2)^2 + (4a + b + 6)^2$

(c) $E(a, b) = (a + b - 5)^2 + (3a + b + 2)^2 + (4a + b - 6)^2$

(d) $E(a, b) = (a + b - 5)^2 + (2a + b + 2)^2 + (3a + b - 6)^2$

(e) $E(a, b) = (a + b + 5)^2 + (3a + b - 2)^2 + (4a + b + 6)^2$

2. (5 pts.) Find the second partial derivative $\dfrac{\partial^2 f}{\partial y \partial x}$ if $f(x, y) = \ln(3x + 2y)$.

(a) $\dfrac{-9}{(3x + 2y)^2}$ (b) $\dfrac{-6}{(3x + 2y)^2}$ (c) $\dfrac{-4}{(3x + 2y)^2}$

(d) $\dfrac{4}{(3x + 2y)^2}$ (e) $\dfrac{6}{(3x + 2y)^2}$

3. (5 pts.) Give the equation for the y-section at $y = 3$ of the surface $z = y^3 - x^2y - 5$.

(a) $z = -6x$ (b) $y^3 - x^2y = 8$ (c) $z = 22 - 3x^2$

(d) $z = 27 - x^2$ (e) None of these.

4. (5 pts.) You wish to find the rectangular box that has the sum of the length x, width y, and the height z equal to 80 centimeters, and maximum volume.

Which of the following systems needs to be solved when using the method of Lagrange Multipliers.

(a) $yz = \lambda, \quad xz = \lambda, \quad xy = \lambda, \quad x + y + z - 80 = 0$

(b) $yz = \lambda, \quad xz = \lambda, \quad xy = \lambda, \quad xyz - 80 = 0$

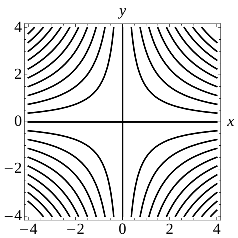
(c) $1 = \lambda yz, \quad 1 = \lambda xz, \quad 1 = \lambda xy, \quad xyz - 80 = 0$

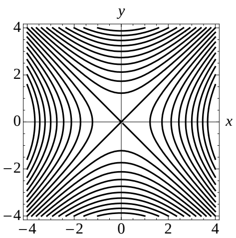
(d) $yz = 1, \quad xz = 1, \quad xy = 1, \quad x + y + z - 80 = 0$

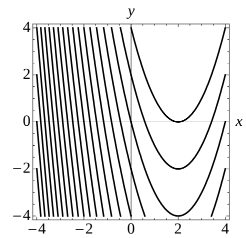
(e) $yz = 1, \quad xz = 1, \quad xy = 1, \quad xyz - 80 = 0$

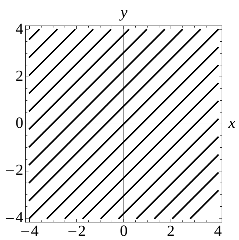
5. (5 pts.) Which of the following is the graph of level curves of the function

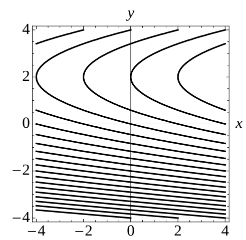
$$f(x,y) = x - (y-2)^2 \ ?$$

(a) 

(b) 

(c) 

(d) 

(e) 

6. (5 pts.) Let $f(x,y) = e^{x^2 y}$. Find the value of the limit

$$\lim_{h\to 0} \frac{f(2,-1+h) - f(2,-1)}{h} \ .$$

(a) $2e^{-2}$ (b) $4e^{-4}$ (c) e^{-4}

(d) $-2e^{-2}$ (e) Does not exist.

7. (5 pts.) The production of a certain company is given by the function $P(K, L)$ where K is the amount of capital and L is the amount of labor. If 2 units of capital and 3 units of labor give 8 units of production, and $MPK(2, 3) = 1$ and $MPL(2, 3) = 4$, find the linear approximation of P at $(2, 3)$.

(a) $P(K, L) \approx 4K + L - 11$

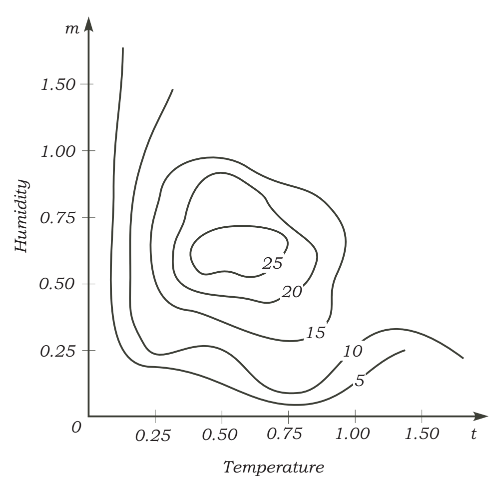
(b) Cannot be determined.

(c) $P(K, L) \approx 4K + L + 8$

(d) $P(K, L) \approx K + 4L - 14$

(e) $P(K, L) \approx K + 4L - 6$

8. (5 pts.) The growth rate $g(t, m)$ of a bacteria in the laboratory depends on the temperature t and humidity m of its environment. Level curves of $g(t, m)$ are given below. What is the maximum growth rate if the equipment in the laboratory could only produce temperature and humidity such that $2t + m = 1$?



(a) 20 (b) 15 (c) Cannot be determined.

(d) 25 (e) 10

9. (5 pts.) Which of the following statement or statements below are **FALSE**?

(1) The surface $x = 5$ is a plane that is perpendicular to the x-axis.

(2) The x–slope of the plane $z = ax + by + c$ is b/a.

(3) $(-3, 2, 1)$ is a point on the plane $z = -(x - 3) + 2(y + 2) + 1$.

(a) (1) and (3) only (b) (2) and (3) only (c) (3) only

(d) (2) only (e) None is false.

10. (5 pts.) It is known that $(1, 1)$ is a critical point of $f(x, y)$ and that

$$\frac{\partial^2 f}{\partial x^2} = 2y - 3, \qquad \frac{\partial^2 f}{\partial y^2} = 2 + 2y, \qquad \frac{\partial^2 f}{\partial x \partial y} = 2x - 3$$

Determine the nature of the critical point $(1, 1)$.

(a) Local maximum. (b) Saddle point. (c) Local minimum.

(d) Global minimum. (e) Cannot be determined.

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) Consider the function of two variables $f(x, y)$. Suppose that

$$\frac{\partial f}{\partial x} = -y + \frac{8}{x^2} \qquad \text{and} \qquad \frac{\partial f}{\partial y} = -x + \frac{8}{y^2} \ .$$

(a) Compute all second-order partial derivatives of $f(x, y)$.

(b) Give the determinant function $D(x, y)$.

(c) It is known that $(2, 2)$ is a critical point of f. Use the second derivative test to determine the nature of the critical point $(2, 2)$.

12. (10 pts.) A pharmaceutical company sells flu medicine in packages of six pills and packages of twelve pills. Suppose x is the number of six-pill-packages and y is the number of twelve-pill-packages that are produced each month. The demand function for the six-pill-packages is $p_1 = 60 - 2x + y$ and the one for the twelve-pill-packages is $p_2 = 10 - x + 2y$ (measured in dollars per package). Answer the following questions regarding this company.

(Part A) Find the monthly revenue function $R(x, y)$.

(Part B) Suppose we also know that each six-pill-package costs \$15 to produce, each twelve-pill-package costs \$25 to produce, and the fixed cost for the monthly total production is \$5000. What is the monthly cost function $C(x, y)$?

(Part C) Find the monthly profit function $P(x, y)$.

13. (10 pts.) Find all critical points of the function

$$f(x, y) = x^3 + 3xy - 3y^2.$$

14. (10 pts.) Find the equation of the plane passing through the points

$$(1, 1, -2); \qquad (2, 2, 1); \qquad (1, 2, -1)$$

15. (10 pts.) The total production of a company manufacturing bubble gum is given by the function

$$f(x, y) = 3x^{1/3}y^{2/3}$$

where x is the number of units of labor and y is the number of units of capital. Suppose each unit of labor costs \$50, each unit of capital costs \$40, and the company's total budget for labor and capital is \$60,000. Use the method of Lagrange multipliers to find the labor and capital that maximizes the output.

Department of Mathematics
University of Notre Dame
Math 10260 – Bus. Calc. 2
Spring 2008

Name:__________________________

Instructor:________________________

Exam II

March 13, 2008

This exam is in 2 parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. May the force be with you.

You must record here your answers to the multiple choice problems.
Place an × through your answer to each problem.

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(•)	(c)	(d)	(e)
3.	(a)	(b)	(•)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(•)
6.	(a)	(•)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(•)	(c)	(d)	(e)
10.	(a)	(•)	(c)	(d)	(e)

MC.	______
11.	______
12.	______
13.	______
14.	______
15.	______
Tot.	______

Math 10260 Exam II (Bus. Calc. 2) Solution – Spring 2008

1. $E(a,b) = (a+b-5)^2 + (3a+b+2)^2 + (4a+b-6)^2$.

2.

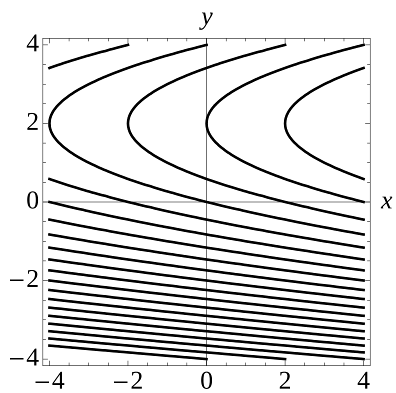
$$\frac{\partial f}{\partial x} = \frac{3}{3x+2y}; \quad \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{3}{3x+2y}\right) = \frac{-6}{(3x+2y)^2}$$

3. $z = 3^3 - x^2 \cdot 3 - 5 = 22 - 3x^2$.

4. We need to maximize the function xyz subject to the constraint $z + y + z - 80 = 0$. The system is:

$$yz = \lambda, \quad xz = \lambda, \quad xy = \lambda, \quad x + y + z - 80 = 0.$$

5. The level curves are $x - (y-2)^2 = c \Rightarrow x = (y-2)^2 + c$. Therefore the answer is

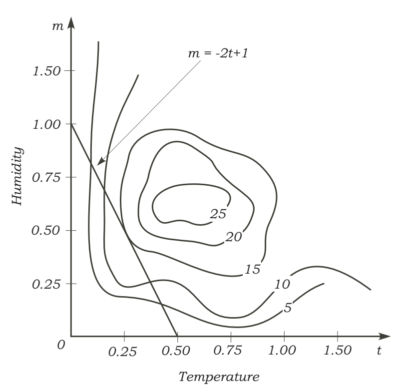


6. The limit is equal to $\frac{\partial f}{\partial y}(2,-1)$.

$$\frac{\partial f}{\partial y} = x^2 e^{x^2 y} \text{ and } \frac{\partial f}{\partial y}(2,-1) = 2^2 e^{-2^2} = 4e^{-4}.$$

7. The linear approximation is:
$P(K,L) = P(2,3) + MPK(2,3)(K-2) + MPL(2,3)(L-3) = 8 + (K-2) + 4(L-3) = K + 4L - 6$

8.



The line $m = -2t + 1$ is tangent to the level curve $z = 15$, and the maximum growth rate is 15.

9. (1) is TRUE. (2) is FALSE: the x-slope is a. (3) is FALSE since $1 \neq -(-3-3) + 2(2+2) + 1 = 15$.

10. Since $D(1,1) = (2 \cdot 1 - 3)(2 + 2 \cdot 1) - (2 \cdot 1 - 3)^2 < 0$, the point $(1,1)$ is a saddle point.

11. (a) $\frac{\partial^2 f}{\partial x^2} = -\frac{16}{x^3}$, $\frac{\partial^2 f}{\partial y \partial x} = -1$, $\frac{\partial^2 f}{\partial y^2} = -\frac{16}{y^3}$, $\frac{\partial^2 f}{\partial x \partial y} = -1$.

(b) $D(x,y) = \left(-\frac{16}{x^3}\right) \cdot \left(-\frac{16}{y^3}\right) - 1^2 = \frac{256}{x^3 y^3} - 1$

(c) Since $D(2,2) = \frac{256}{2^3 \cdot 2^3} - 1 = 3 > 0$ and $\frac{\partial^2 f}{\partial x^2}(2,2) = -\frac{16}{2^3} = -2 < 0$, the function f has a local maximum at $(2,2)$.

12. **Part A.** $R(x,y) = xp_1 + yp_2 = x(60 - 2x + y) + y(10 - x + 2y) = 60x + 10y - 2x^2 + 2y^2$ dollars.

Part B. $C(x,y) = 15x + 25y + 5000$ dollars.

Part C. $P(x,y) = R(x,y) - C(x,y) = 60x + 10y - 2x^2 + 2y^2 - 15x - 25y - 5000 =$ $45x - 15y - 2x^2 + 2y^2 - 5000$ dollars.

13. $\frac{\partial f}{\partial x} = 3x^2 + 3y$, and $\frac{\partial f}{\partial y} = 3x - 6y$. We need to solve the system

$$\begin{cases} 3x^2 + 3y = 0 \\ 3x - 6y = 0 \end{cases}$$

From the second equation we obtain $y = \frac{1}{2}x$. Combining it with the first equation we have $3x^2 + \frac{3}{2}x = 0$. This equation has two solutions: $x = 0$ and $x = -1/2$.
Critical points: $(0,0)$ and $(-1/2, -1/4)$.

14. Using the "point-slopes" formula $z - z_0 = a(x - x_0) + b(y - y_0)$ with the first point as (x_0, y_0, z_0), we obtain the equation $z + 2 = a(x - 1) + b(y - 1)$ with unknown slopes a and b. Since the second and third points are on the plane, we have two equations $1 + 2 = a(2 - 1) + b(2 - 1)$ and $-1 + 2 = a(1 - 1) + b(2 - 1)$. Hence $b = 1$ and $a = 2$.
An equation of the plane: $z + 2 = 2(x - 1) + (y - 1)$.

15. We need to maximize the function $f(x,y) = 3x^{1/3}y^{2/3}$ subject to the constraint $g(x,y) = 50x + 40y - 60000 = 0$.
We have $\frac{\partial f}{\partial x} = x^{-2/3}y^{2/3}$, $\frac{\partial g}{\partial x} = 50$, $\frac{\partial f}{\partial y} = 2x^{1/3}y^{-1/3}$, and $\frac{\partial g}{\partial x} = 40$.
Thus we need to solve the system

$$\begin{cases} x^{-2/3}y^{2/3} = \lambda \cdot 50 \\ 2x^{1/3}y^{-1/3} = \lambda \cdot 40 \\ 50x + 40y - 60000 = 0 \end{cases}$$

From the first two equations we obtain $\frac{1}{50}x^{-2/3}y^{2/3} = \frac{2}{40}x^{1/3}y^{-1/3}$, which reduces to $4y = 10x$, or $y = \frac{5}{2}x$. By substituting this into the third equation of our system we get $150x = 60000$, or $x = 400$. And it follows that $y = 1000$.
Answer: $x = 400, y = 1000$.

Department of Mathematics
University of Notre Dame
Math 10260 – Bus. Calc. 2
Spring 2008

Name:______________________________

Instructor:___________________________

Exam III

April 15, 2008

This exam is in 2 parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. May the force be with you.

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature:______________________________

You must record here your answers to the multiple choice problems.
Place an × through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC.	________
11.	________
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14.	________
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Tot.	________

Multiple Choice

1. (5 pts.) The capital stock $k(t)$ for an economy is given by

$$\frac{dk}{dt} = 2\sqrt{k} - k; \qquad k(0) = 1$$

According to the model, what value does the capital stock approach after a long time?

(a) 2

(b) 0

(c) Cannot be determined.

(d) 4

(e) $+\infty$

2. (5 pts.) Evaluate the following geometric series

$$\frac{\pi}{2} + \frac{\pi^2}{4} + \frac{\pi^3}{8} + \frac{\pi^4}{16} + \cdots$$

(a) $\dfrac{\pi}{1+\pi}$

(b) $\dfrac{\pi}{2+\pi}$

(c) Series is divergent.

(d) $\dfrac{\pi}{2-\pi}$

(e) $\dfrac{\pi}{1-\pi}$

3. (5 pts.) Suppose you roll a pair of fair dice. Find the probability that the sum of the top faces is less than or equal to 4.

(a) 5/36 (b) 6/36 (c) 4/36

(d) 3/36 (e) 2/36

4. (5 pts.) Let $q = f(x)$ be the **demand function** of an ipod accessory. Here q is the quantity sold in thousands and x is the price in tens of dollars. Suppose that

$$f(2) = 5; \quad f'(2) = -1; \quad f''(2) = 3$$

Compute the 2nd-degree Taylor polynomial for the **revenue** $R(x)$ about 2.

(a) $P_2(x) = 10 + 3x + 2x^2$

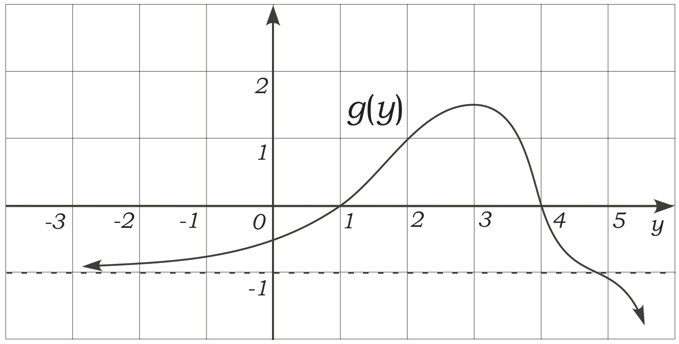
(b) $P_2(x) = 10 + 3(x-2) + 4(x-2)^2$

(c) $P_2(x) = 10 + 3(x-2) + 2(x-2)^2$

(d) $P_2(x) = 10 + 3x + 4x^2$

(e) $P_2(x) = 10 + 3(x+2) + 4(x+2)^2$

5. (5 pts.)



The size $y(t)$ of a population (in hundreds) of wild crocodile grows according to the model

$$\frac{dy}{dt} = g(y)$$

where the graph of $g(y)$ is given above. Find the **equation** of the **asymptotically stable** equilibrium solution of the differential equation.

(a) $y = 3$ (b) None of these. (c) $y = -1$

(d) $y = 1$ (e) $y = 4$

6. (5 pts.) For the **same** population of crocodile above, which of the following statements are **TRUE**?

(1) If $y(0) = 1.5$ then the size of the population approaches 3 as $t \to \infty$.

(2) The carrying capacity of the environment is 4 hundred crocodiles.

(3) If $y(0) = 2$ then the population is decreasing in size as time progresses.

(4) If $y(0) = 0.5$ then the population is headed for extinction.

(a) (1), (2) and (4) only (b) (4) only (c) (3) and (4) only

(d) (2) and (4) only (e) (3) only

7. (5 pts.) Suppose that the 6th-degree Taylor polynomial of $f(x)$ about -3 is

$$P_6(x) = 5 - 2(x+3)^3 - 4(x+3)^4 + (x+3)^6.$$

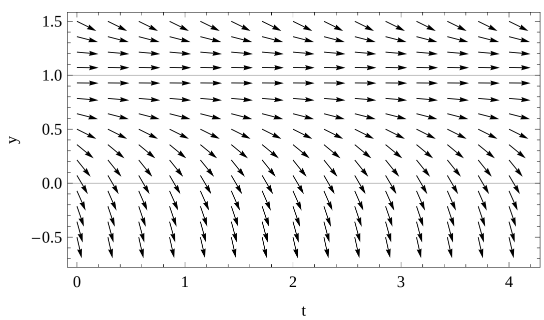
What is the value of $f'''(-3)$?

(a) -12 (b) 24 (c) -24

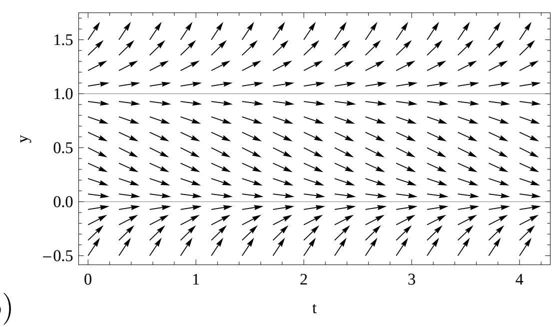
(d) 12 (e) -48

8. (5 pts.) Which of the pictures below shows the direction field of the differential equation

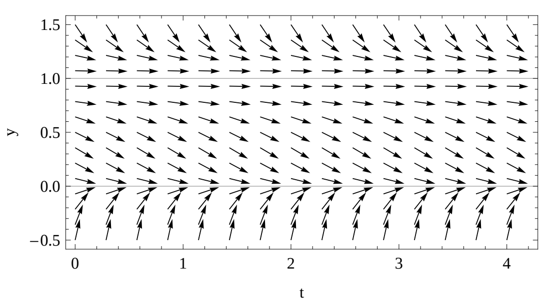
$$\frac{dy}{dt} = -2y(y-1)?$$



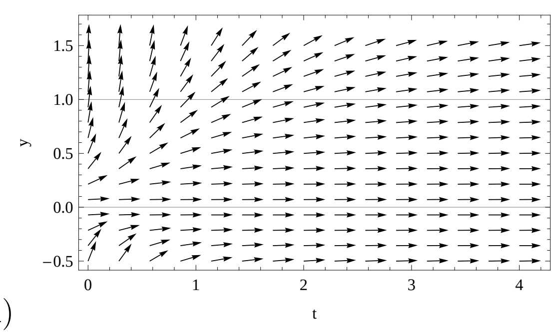
(a)



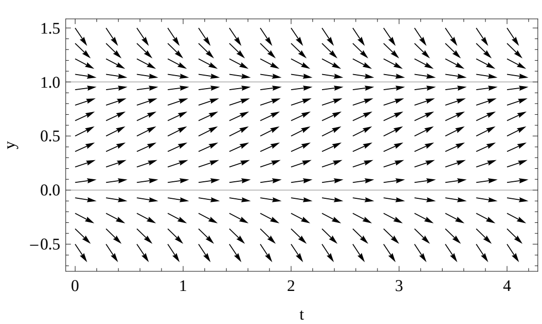
(b)



(c)



(d)



(e)

9. (5 pts.) Which of the following expression is the 3^{rd} degree Taylor polynomial for $\ln(2+x)$ about -1?

(a) $(x-1)-(x-1)^2+2(x-1)^3$

(b) $(x+1)-(x+1)^2+2(x+1)^3$

(c) $x-\dfrac{x^2}{2}+\dfrac{x^3}{3}$

(d) $(x-1)-\dfrac{(x-1)^2}{2}+\dfrac{(x-1)^3}{3}$

(e) $(x+1)-\dfrac{(x+1)^2}{2}+\dfrac{(x+1)^3}{3}$

10. (5 pts.) The table below gives the probability for the selection of a customer who bought a cookie from a cafe. Assuming that the possible cookies carried by the cafe are listed below, find the value of a.

Selections	Chocolate	Sugar	Peanut Butter	Oatmeal	Gingerbread
Probability	0.30	$2a$	0.25	0.15	a

(a) 0.3 (b) Cannot be determined. (c) 0.1

(d) 1 (e) 0.15

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) The number of cellphones (in millions) owned by the residents of a city is given by the equation

$$\frac{dy}{dt} = ye^{-t}; \qquad y(0) = 2$$

where t is the time in years from 2006. Use Euler's method with $\Delta t = 1$ to estimate the number of cellphones owned in the year **2008**.

12. (10 pts.) Consider the differential equation

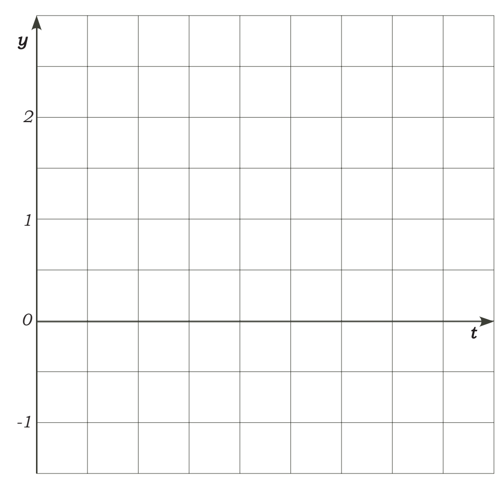
$$\frac{dy}{dt} = y^2 - 2y$$

(a) Find the **equation(s)** of all equilibrium solution(s) of the differential equation.

(b) Find the **equation(s)** of all inflection line(s).

(c) On the grid below, mark **all** equilibrium solutions and inflection lines. Then, sketch the solution curve for the initial value problem below. Be sure to **show clearly the concavity** of the solution curve.

$$\frac{dy}{dt} = y^2 - 2y; \qquad y(0) = 1.5$$



13. (10 pts.) Let $y(t)$ be the solution to the initial value problem

$$y' = 2e^y + t, \qquad y(2) = 0.$$

(a) Compute the 2nd-degree Taylor polynomial for $y(t)$ about 2.

(b) Use your result in Part (a) to estimate $y(2.2)$.

14. (10 pts.) Suppose you want to a establish a **perpetual** fund that pays out a certain amount at the end of every year, starting with \$1,000 at end of the first year and increasing the amount by 2% every year. Assuming that your investment pays 4% annual interest, compounded **continuously**, answer the following questions:

(a) What is the **present value** of the first payment? ________________________.

(b) What is the **present value** of the second payment? ________________________.

(c) Write down the geometric series that gives the value of the lump sum you must invest to establish the **perpetual** fund. Also give the common ratio of your series. **You must give at least the first three terms of the series in your answer.**

Common ratio of series = ____________________

(d) What is value of the lump sum you must invest?

15. (10 pts.) A study of caucasians showed that 40% had blond hair, 30% had blue eyes, and 20% had blond hair and blue eyes . Find the probabilities below.

(Hint: Let E be the event that a caucasian had blond hair, and G be the event that a caucasian had blue eyes.)

a. Probability that a caucasian **did not** have blonde hair.

b. Probability that a caucasian had blonde hair **or** blue eyes (or both).

c. Probability that a caucasian had blonde hair **given that** he/she had blue eyes.

Department of Mathematics
University of Notre Dame
Math 10260 – Bus. Calc. 2
Spring 2008

Name:________________________________

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Exam III

April 15, 2008

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3.	(a)	(•)	(c)	(d)	(e)
4.	(a)	(b)	(•)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(•)
6.	(a)	(b)	(c)	(•)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(•)
9.	(a)	(b)	(c)	(d)	(•)
10.	(a)	(b)	(•)	(d)	(e)

MC.	______
11.	______
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13.	______
14.	______
15.	______
Tot.	______

10260 Bus. Calc. 2
April 2008
Solutions: Exam III

1. The equilibria values are $k = 0$ (unstable) and $k = 4$ (stable), so the capital stock approaches 4 after a long time. The answer is (d.)

2.

$$\frac{\pi}{2} + \frac{\pi^2}{4} + \frac{\pi^3}{8} + \ldots = \frac{\pi}{2}\left(1 + \frac{\pi}{2} + (\frac{\pi}{2})^2 + (\frac{\pi}{2})^3 + \ldots\right)$$

The geometric series diverges since $\pi/2$ is > 1. The correct answer is (c).

3. There are 36 equally likely outcomes: $(1,1), (1,2), \ldots, (6,6)$. The event E that the sum of top faces is less than or equal to 4 consists of 6 outcomes: $(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)$. Hence $P(E) = 6/36$. The correct answer is (b).

4. $R(x) = xq = xf(x)$. The degree 2 Taylor polynomial of $R(x)$ about 2 is

$$P_2(x) = R(2) + \frac{R'(2)}{1!}(x-2) + \frac{R''(2)}{2!}(x-2)^2.$$

We have $R(2) = 2 \cdot f(2) = 2 \cdot 5 = 10$.

For the other coefficients in $P_2(x)$, note that $R'(x) = [xf(x)]' = f(x) + xf'(x)$. Thus $R'(2) = f(2) + 2f'(2) = 5 + 2 \cdot (-1) = 3$. Then $R''(x) = [f(x) + xf'(x)]' = f'(x) + f'(x) + xf''(x)$, and so $R''(2) = 2f'(2) + 2f''(2) = -2 + 2 \cdot 3 = 4$. Plugging these into P_2 we get that the correct answer is (c).

5. Since $g(4) = 0$ and $g'(4) < 0$, $y = 4$ is the equation of the asymptotically stable equilibrium solution. The answer is (e).

6. If $y(0) = 1.5$, then the population approaches 4 as $t \to \infty$ (see preceding problem), so (1) is FALSE. Again, by the previous problem, (2) is TRUE, while (3) is FALSE. Since $y = 1$ is an unstable equilibrium solution, (4) is TRUE. Therefore, the answer is (d).

7. The coefficient of $(x+3)^3$ in $P_6(x)$ has to equal $\frac{f'''(-3)}{3!}$. Hence $f'''(-3) = (-2) \cdot 3! = -12$. The correct answer is (a).

8. Setting $-2y(y-1) = 0$, we see there are equilibrium solutions at $y = 0$ (unstable) and $y = 1$ (stable). This rules out every picture except (e.) (just test different initial values: less than 0, between 0 and 1, and greater than 1).

9. Let $f(x) = \ln(2+x)$. The degree 3 Taylor polynomial of $f(x)$ about -1 is

$$P_3(x) = f(-1) + \frac{f'(-1)}{1!}(x+1) + \frac{f''(-1)}{2!}(x+1)^2 + \frac{f'''(-1)}{3!}(x+1)^3.$$

We have $f(-1) = \ln 1 = 0$. It is necessary then that $P_3(-1) = f(-1) = 0$. Thus (a), (c), and (d) are not correct answers. We need to decide among (b) and (e). Note that $f'(x) = \frac{1}{2+x}$ and so $f'(-1) = 1$. So far, any of (b) and (e) can be correct. Let's look at the coefficient of $(x+1)^2$. Note that $f''(x) = \frac{-1}{(2+x)^2}$ and so $f''(-1) = -1$. Thus only (e) can be the correct answer. To check this is indeed the correct answer, we only have to check that the coefficient of $(x+1)^3$ is ok. Note that $f'''(x) = \frac{2}{(2+x)^3}$ and so $f'''(-1) = 2$. Thus the coefficient of $(x+1)^3$ is indeed $2/3! = 2$.

10. $1 = 0.3 + 2a + 0.25 + 0.15 + a$, hence $0.3 = 3a$, and $a = 0.1$. The correct answer is (c).

11. Since $\Delta t = 1$, we have $n = 2$; i.e., $t_0 = 0, t_1 = 1, t_2 = 2$. Let

$$f(t,y) = ye^{-t}. \tag{1}$$

Now,

$$\begin{aligned} y_0 &= 2, \\ y_1 &= y_0 + f(t_0, y_0)\Delta t \\ &= 2 + f(0,2) \cdot 1. \end{aligned}$$

Using (1),

$$f(0,2) = 2e^0 = 2.$$

Therefore,

$$y_1 = 2 + 2 = 4.$$

Also,

$$\begin{aligned} y_2 &= y_1 + f(t_1, y_1)\Delta t \\ &= 4 + f(1,4) \cdot 1. \end{aligned}$$

Again using (1),

$$f(1,4) = 4e^{-1}.$$

So

$$y(2) \approx y_2 = 4 + 4e^{-1}.$$

12. (a.) Set $g(y) = y^2 - 2y = y(y-2) = 0$, then we have equilibrium solutions $y = 0$ (stable) and $y = 2$ (unstable).

(b.) Since $y'' = g'(y)g(y) = 2y(y-1)(y-2)$, the inflection lines are at $y = 0$, $y = 1$, and $y = 2$.
(c.) Since the initial condition $0 < y(0) < 2$, we know the solution curve will (i) start at the point $(0, 1.5)$; (ii) be decreasing with $y(t) \to \infty$ as $t \to \infty$; (iii) be concave *down* for $1 < y(t) < 1.5$, and (iv) concave *up* for $0 < y(t) < 1$.

13. (a)

$$P_2(t) = y(2) + \frac{y'(2)}{1!}(t-2) + \frac{y''(2)}{2!}(t-2)^2.$$

It is given that $y(2) = 0$. Also, $y'(2) = 2e^{y(2)} + 2 = 2 \cdot 1 + 2 = 4$. Note that $y'' = 2e^y y' + 1$ and so $y''(2) = 2e^{y(2)}y'(2) + 1 = 2 \cdot 4 + 1 = 9$. Hence

$$P_2(t) = 4(t-2) + 4.5(t-2)^2.$$

(b) $y(2.2) \approx P_2(2.2) = 4(0.2) + 4.5(0.2)^2 = 0.98$

14. (a) $1000 = PVe^{0.04}$, so $PV = 1000e^{-0.04}$.
(b) $1000(1.02) = PVe^{(0.04)2}$, so $PV = 1000(1.02)e^{-(0.04)2}$.
(c) $1000e^{-0.04} + 1000(1.02)e^{-(0.04)2} + 1000(1.02)^2e^{-(0.04)3} + \ldots$
(d) The sum from (c) equals

$$1000e^{-0.04}\left[1 + \frac{1.02}{e^{0.04}} + \left(\frac{1.02}{e^{0.04}}\right)^2 + \left(\frac{1.02}{e^{0.04}}\right)^3\right] + \ldots =$$

$$= 1000e^{-0.04}\frac{1}{1 - \frac{1.02}{e^{0.04}}}$$

$$= 48052.$$

15. We are told that $P(E) = .4, P(G) = .3, P(E \cap G) = .2$.
(a.) $P(E') = 1 - P(E) = 1 - .4 = .6$
(b.) $P(E \cup G) = P(E) + P(G) - P(E \cap G) = .4 + .3 - .2 = .5$.
(c.) $P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{.2}{.3} = .66$