

Name: _____

Instructor: _____

Exam III

April 16, 2009

This exam is in 2 parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature: _____

You must record here your answers to the multiple choice problems.

Place an \times through your answer to each problem.

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|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
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Multiple Choice

1. (5 pts.) Prior to the collapse of the investment bank Bear Stearns, it was revealed that one of its investment portfolios, the Alt-A trust, consisted of 2093 mortgage loans.¹ At that time, 195 of those loans were in foreclosure and 180 were delinquent. What was the probability that a randomly selected loan from this fund was in foreclosure or delinquent? (Note: A loan is either in foreclosure or delinquent, but not both. Answers below are rounded to the nearest thousandth.)

- (a) 0.093
- (b) 0.086
- (c) 0.179
- (d) 0.171
- (e) None of these.

2. (5 pts.) The fourth degree Taylor polynomial of $f(x)$ about 2 is

$$P_4(x) = 1 + 2(x - 2) + 3(x - 2)^3 + 4(x - 2)^4.$$

What is $f''(2)$?

- (a) None of these
- (b) 3
- (c) 18
- (d) $\frac{1}{3!}$
- (e) 6

¹Source: New York Times, V. Bajaj, "Plan's Mystery: What's all this stuff worth?" Sept. 2008

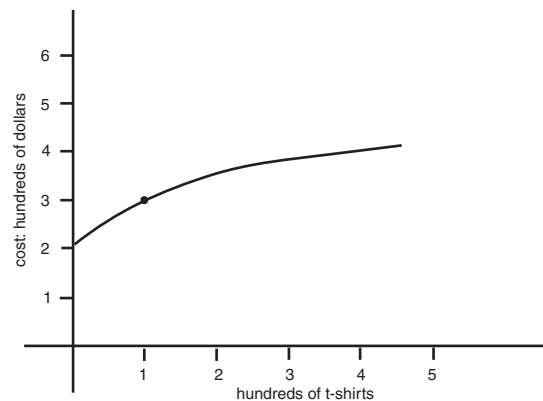
3. (5 pts.) A treatment is being designed in which a patient is injected daily with a dose of A mg of a new drug. The drug is eliminated exponentially from the body with any single injection leaving $Ae^{-0.2t}$ mg remaining in the body at the end of the day, t days after the injection. Over a long period of treatment, the build up of drug in the body could reach the toxic level of 1000 mg if the dosage A is not chosen correctly. If the treatment is to be continued indefinitely, which of the following dosage levels is the largest possible that is still safe to use?

(Hint: Set up an infinite geometric series to approximate the amount of drug in the patient after a long time assuming that measurements are made immediately after injection.)

- (a) $A = 150$ mg
- (b) $A = 0$ mg
- (c) $A = 250$ mg
- (d) $A = 100$ mg
- (e) $A = 180$ mg

4. (5 pts.) The cost $C(x)$, in hundreds of dollars, of producing x hundred t-shirts has the graph shown below. Which of the following functions could be the second degree Taylor polynomial of $C(x)$ about $x = 1$?

- (a) $P_2(x) = 2 + x - 2x^2$
- (b) $P_2(x) = 3 + 0.7(x - 1) - 0.25(x - 1)^2$
- (c) $P_2(x) = 3 - 2(x - 1) + (x - 1)^2$
- (d) $P_2(x) = 2 + 0.7(x - 1) - 0.25(x - 1)^2$
- (e) $P_2(x) = 3 + 0.7(x - 1) + 0.25(x - 1)^3$



5. (5 pts.) Consider the series

$$\sum_{k=0}^{\infty} 5 \left(-\frac{1}{3}\right)^k = 5 - \frac{5}{3} + \frac{5}{9} - \frac{5}{27} + \dots$$

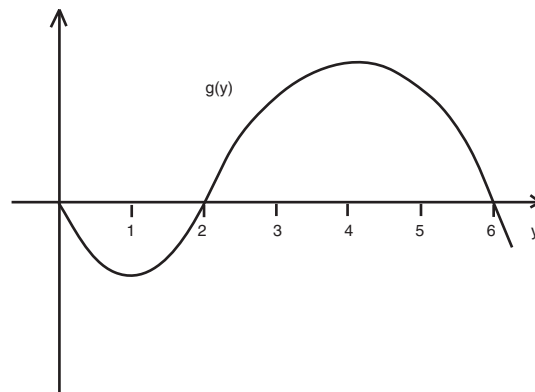
and select the correct statement.

- (a) The series is convergent and the sum is $\frac{4}{15}$.
- (b) The series is convergent and the sum is $\frac{15}{2}$.
- (c) The series is divergent.
- (d) The series is convergent and the sum is $\frac{15}{4}$.
- (e) The series is convergent and the sum is $\frac{3}{4}$.

6. (5 pts.) The population y of unicorns (in hundreds) changes according to the model

$$\frac{dy}{dt} = g(y)$$

where $g(y)$ has the graph shown in the figure and the variable t is the number of years since the ancient time when precisely 100 unicorns roamed. Which of the following statements is **false**?

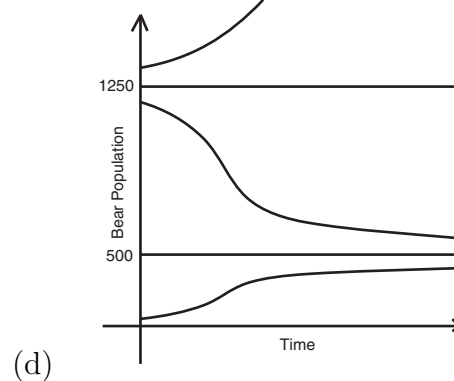
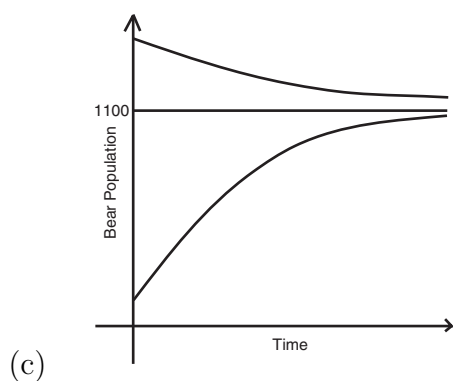
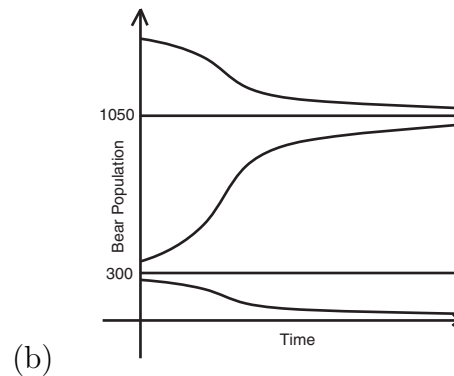
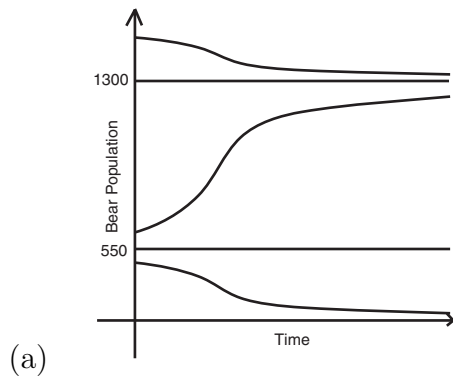


- (a) The graph of the solution with initial value 3 must cross an inflection line.
- (b) With $y(0) = 1$ as described above, $\lim_{t \rightarrow \infty} y(t) = 0$. That is, the unicorns were doomed to extinction.
- (c) If the initial population had been greater than 600, then $\lim_{t \rightarrow \infty} y(t) = \infty$. That is, the population would have increased without limit.
- (d) The equilibrium solution $y = 2$ is unstable.
- (e) If the initial population had been more than 200 unicorns, we would have $\lim_{t \rightarrow \infty} y(t) = 6$. That is, the population would stabilize at 600 unicorns.

7. (5 pts.) An urn contains 6 red balls and 4 green balls. Two balls are drawn from the urn without replacement. What is the probability that the second ball drawn is red?

- (a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{3}$ (d) $\frac{7}{15}$ (e) $\frac{4}{15}$

8. (5 pts.) Four members of your team propose differential equations modeling the population $p(t)$ of bears whose territory is confined to a large forest. The sketches of solutions for each of their models are shown below. Which model should you reject immediately?



(e) None of these. Each model is feasible.

9. (5 pts.) An experiment with outcomes $x_1, x_2, x_3, x_4,$ and x_5 is described by the probability table below.

Outcome	x_1	x_2	x_3	x_4	x_5
Probability	0.15	p	0.25	$2p$	0.3

What is the value of p ?

- (a) 0.30 (b) 0.15 (c) 0.10 (d) -0.15 (e) None of these.

10. (5 pts.) Let X be a random variable with range $\{20, 30, 50\}$ and probability distribution $P(X = 20) = 0.25$, $P(X = 30) = 0.25$, and $P(X = 50) = 0.5$. What is the expected value of X ?

- (a) 40 (b) 30 (c) $\frac{100}{3}$ (d) 35 (e) $\frac{75}{2}$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) Let $y(t)$ be the solution to the initial value problem

$$\frac{dy}{dt} = e^y, \quad y(0) = 0.$$

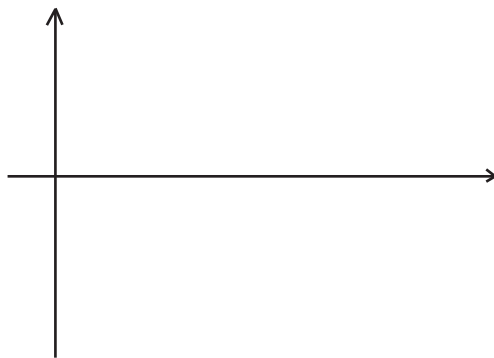
Compute the 3rd degree Taylor polynomial of $y(t)$ about $t = 0$.

12. (10 pts.) Parts A. and B. of this problem concern the differential equation

$$\frac{dy}{dt} = 4 - y^2.$$

Part A. Find the equilibrium solutions of this equation and determine their stability.

Part B. Sketch the solution of this equation satisfying the initial condition $y(0) = -0.5$ on the axes supplied below. Indicate the equilibrium solutions, any lines of inflection, and where the solution is concave up and concave down in the figure.



13. (10 pts.) After an immoderately successful business career, your boss wants to endow a scholarship fund at his alma mater, Old Siwash. He wants the fund to pay out \$10,000 at the end of the first year and increase by 4% each subsequent year. He expects the endowment of the fund to earn 6% annual interest, compounded continuously and he wants the fund to last forever. Unable to figure out how much the endowment should be, he turns to you, a Notre Dame graduate. What will you tell him? (Your retention bonus depends on this!)

Part A. Determine the present value of the 10th payment.

Part B. Write an infinite geometric series whose sum gives the value of the initial endowment and compute its sum. How much should your boss endow?

14. (10 pts.) As of the end of March 2009, 8.1% of men were unemployed, whereas 7% of women were unemployed.² The Bureau of Labor Statistics reports that 50% of the potential workforce is male.

Let M be the event that a randomly selected member of the potential workforce is male and let A be the event that a randomly selected member of the potential workforce is unemployed.

Part A. Identify $P(A|M)$, $P(A|M')$ and $P(M)$.

Part B. Compute $P(A)$.

Part C. Compute $P(M|A)$ and state in words what it means.

²Source: New York Times, J. Healy, "Jobless rate hits 8.5% as March payrolls fall by 663,000."

15. (10 pts.)

Part A. Let $f(x) = \frac{1}{x}$. Find the Taylor polynomial of degree 3 of $f(x)$ about $x = 1$.

Part B. Estimate the value of the integral $\int_0^{0.25} e^{-x^2} dx$ using the fact that

$$P_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{6}x^4$$

is the 4th degree Taylor polynomial of $f(x) = e^{-x^2}$ about $x = 0$.

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