

Department of Mathematics
University of Notre Dame
Math 10260 – Bus. Calc. II
Fall 2009

Name: _____

Instructor: _____

Exam II

November 19, 2009

This exam is in 2 parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

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You must record here your answers to the multiple choice problems.

Place an \times through your answer to each problem.

- | | | | | | |
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Multiple Choice

1. (5 pts.) Find all the equilibrium solutions of: $dy/dt = 0.3ye^{1-(y/5)} - 0.6y$.

(a) $y = 0$ and $y = 5$

(b) $y = 0$

(c) $y = 5$ and $y = \ln 2$

(d) $y = 0$ and $y = 5 - 5 \ln 2$

(e) $y = 0$, $y = 1 - \ln 2$, and $y = \ln 5$

2. (5 pts.) If the population $p(t)$ (in thousands) of a bird species in a region satisfies the differential equation $\frac{dp}{dt} = 0.02p(p-2)(24-p)$, find the minimum number of birds needed to introduce them successfully into another region with identical environment.

(a) 2,000

(b) 1,200

(c) 1,000

(d) 4,000

(e) 2,400

3. (5 pts.) Find the second-degree Taylor polynomial of $f(x) = \ln \frac{4}{x+1}$ about the point $a = 3$.

(a) $(x - 3) + \frac{1}{2!}(x - 3)^2$

(b) $-\frac{1}{4}(x - 3) - \frac{1}{16}(x - 3)^2$

(c) $-\frac{1}{4}(x - 3) - \frac{1}{32}(x - 3)^2$

(d) $-\frac{1}{4}(x - 3) + \frac{1}{16}(x - 3)^2$

(e) $-\frac{1}{4}(x - 3) + \frac{1}{32}(x - 3)^2$

4. (5 pts.) If $y(x)$ solves the initial value problem $dy/dx = 4x^3 + y$, $y(-1) = 6$, then find the second-degree Taylor polynomial of $y(x)$ about the point $a = -1$.

(a) $6 + 2(x - 1) + 7(x - 1)^2$

(b) $6 + 2(x + 1) + 7(x + 1)^2$

(c) $-1 + 2(x + 1) + 4(x + 1)^2$

(d) $2(x + 1) + 4(x + 1)^2$

(e) $6 + 2(x + 1) + 14(x + 1)^2$

5. (5 pts.) Find the sum $\sum_{k=0}^{\infty} 2(1/3)^k$.

- (a) 3.5 (b) 3/2 (c) 6 (d) 3
(e) The series diverges

6. (5 pts.) Which one of the following infinite series converges?

- (a) $\sum_{k=0}^{\infty} (-1)^k$ (b) $\sum_{k=0}^{\infty} (-10/9)^k$ (c) $\sum_{k=0}^{\infty} (2/9)^k$
(d) $\sum_{k=0}^{\infty} (11/3)^k$ (e) $\sum_{k=0}^{\infty} 2^k$

7. (5 pts.) A beverage company that spent \$1 million on advertising on the Super Bowl broadcast estimated that its extra sales t days after the game were given by $s(t) = 0.015e^{-0.01t}$ for $t = 1, 2, \dots$, where $s(t)$ is in millions of dollars. Assuming that went on indefinitely, estimate the company's net profit due to the advertisement (assume that is the total extra sales minus advertisement costs). All answers are in millions of dollars.

- (a) 0.49 (b) $e - 1$ (c) 0.64 (d) $1 - e^{-0.01}$ (e) 0.23

8. (5 pts.) Let $S = \{s_1, s_2, s_3, s_4\}$ be a sample space. Suppose that $P(\{s_1, s_2\}) = 0.4$, $P(\{s_2\}) = 0.1$, and $P(\{s_2, s_3\}) = 0.5$. Compute $P(\{s_4\})$.

- (a) 0.3 (b) 0.4 (c) 0.5 (d) 0.2 (e) 0.1

9. (5 pts.) Among the first year students at a college, 80% take a math course, 70% take a chemistry course, and 90% take either a math course or a chemistry course. What is the probability that a randomly selected first year student takes both a math course and a chemistry course?

- (a) 0.5 (b) 0.7 (c) 0.6 (d) 0.9 (e) 0.3

10. (5 pts.) A random variable X has a probability distribution given by the following table.

x	1	2	4	5
$P(X = x)$	0.1	0.2	0.4	0.3

Compute the expectation $E(X)$.

- (a) 4 (b) 2.4 (c) 1 (d) 3.2 (e) 3.6

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) (**Show your work!**) Let $y(t)$ be the solution to the initial value problem

$$\frac{dy}{dt} = y^3 - 9y^2 + 18y, \quad y(0) = 1.$$

- (i) Find $\lim_{t \rightarrow \infty} y(t)$.
- (ii) Determine the concavity of the graph of $y(t)$.
- (iii) Sketch the graph of $y(t)$.

12. (10 pts.) (**Show your work!**) A patient is injected once a day at 8:00 am with 100 units of a certain drug. Suppose this drug is eliminated exponentially, with any single injection leaving an amount of $100e^{-0.1t}$ remaining after t days. Assuming the treatment is continued indefinitely, use a geometric series to estimate the number of units in the patient's system at 8:01 am after a very long time.

13. (10 pts.) (**Show your work!**) Suppose you want to endow an annual scholarship by investing an initial amount of capital at an annual interest rate of 5% compounded continuously. How much must you invest to insure that the fund will pay out \$2,000 at the end of the first year and that the payments increase by 3% every year to adjust for inflation?

14. (10 pts.) (**Show your work!**) An auto parts supplier receives spark plugs from two different suppliers, with an equal number coming from each. She knows that 10% of the spark plugs from supplier I are defective and 15% of those coming from supplier II are defective. If a spark plug is defective, what is the probability it comes from supplier I?

15. (10 pts.) (**Show your work!**) In an urn are 2 green balls, 2 red balls, and 5 yellow balls. Two balls are chosen without replacement.

- (i) What is the probability that both balls are yellow?
- (ii) What is the probability that at least one ball is yellow?

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