Midterm 3 Answers - Bus. Calc. II
November 19, 2009

1. Set $0.3ye^{1-(y/5)} - 0.6y = 0$ and solve for y. You get $y = 0$ and $y = 5 - 5\ln 2$.

2. Set $g(p) = 0.02p(p-2)(24-p) = 0$ to find the equilibrium solutions $p = 0, 2, 24$. Taking the derivative of g and plugging in $p = 0, 2, 24$, you get that $g'(0) < 0$, $g'(2) > 0$, and $g'(24) < 0$. Hence $p = 0, 24$ are stable solutions, whereas $p = 2$ is unstable. That is, if population is below $2,000$, it will go extinct.

3. The 2nd-degree Taylor polynomial about $a = 3$ is $f(3) + \frac{f'(3)}{1!}(x-3) + \frac{f''(3)}{2!}(x-3)^2$. Now, $f(3) = 0$. Also $f'(x) = -1/(x+1)$, so $f'(3) = -1/4$. Then $f''(x) = 1/(x+1)^2$, so $f''(3) = 1/16$. Hence we get: $-\frac{1}{4}(x-3) + \frac{1}{32}(x-3)^2$.

4. The 2nd-degree Taylor polynomial of $y(x)$ about the point $a = -1$ is $y(-1) + \frac{y'(-1)}{1!}(x+1) + \frac{y''(-1)}{2!}(x+1)^2$. Now $y(-1) = 6$. Also, $y'(-1) = 4(-1)^3 + 6 = 2$. And $y''(x) = d(4x^3+y)/dt = 12x^2 + y'(x)$, so $y''(-1) = 12 + 2 = 14$. Hence the answer is: $6 + 2(x+1) + 7(x+1)^2$.

5. This is a geometric series with first term 2 and common ratio $1/3$, hence it converges to $2 \cdot \frac{1}{1-(1/3)} = 3$.

6. All are geometric series. Only $\sum_{k=0}^{\infty}(2/9)^k$ has common ratio of absolute value < 1, so it is the only one that is convergent.

7. The answer is $-1 + \sum_{t=1}^{\infty} 0.015e^{-0.01t} = -1 + 0.015e^{-0.01} \cdot \frac{1}{1-e^{-0.01}} = 0.49$.

8. First, note that $P(\{s_1\}) = P(\{s_1, s_2\}) - P(\{s_2\}) = 0.3$. Thus, $P(\{s_4\}) = 1 - P(\{s_1\}) - P(\{s_2, s_3\}) = 0.2$.

9. Let M be the event that a student takes a math course, and C the event that a student takes a chemistry course. Then, $P(M \cap C) = P(M) + P(C) - P(M \cup C) = 0.6$.

10. By definition,

$$E(X) = 1(0.1) + 2(0.2) + 4(0.4) + 5(0.3) = 3.6.$$

11. (i) Set $g(y) = y^3 - 9y^2 + 18y = y(y-6)(y-3) = 0$. The solutions are $y = 0, 3, 6$.. Compute $g'(y) = 3y^2 - 18y + 18$ and plug in to get: $g'(0) > 0$ and $g'(3) < 0$. So $y = 0$ is unstable, but $y = 3$ is stable. Hence $\lim_{t\to\infty} y(t) = 3$.

(ii) Since $g'(y) = 3(y-(3-\sqrt{3}))(y-(3+\sqrt{3}))$, the sign of $g \cdot g'$ is positive for $y \in (0, 3-\sqrt{3})$ and negative for $y \in (3-\sqrt{3}, 3)$. Hence $y(t)$ begins concave up with $y(0) = 1$, until when $y = 3 - \sqrt{3} = 1.27$, from which point it is concave down.

12. This is the sum $100 + 100e^{-0.1} + 100e^{-0.1\cdot 2} + 100e^{-0.1\cdot 3} + \ldots = 100 \cdot \frac{1}{1-e^{-0.1}}$.

13. Summing up the present values of all payments we get: $\sum_{k=1}^{\infty} 2,000(1.03)^{k-1}e^{-0.05k} = 2,000e^{-0.05} \cdot \frac{1}{1-(1.03)e^{-0.05}} = 94,025$.

14. Let E be the event that a spark plug is defective and F the event that a spark plug comes from supplier I; we wish to calculate $P(F|E)$. Since half of the spark plugs come from each supplier, $P(E) = 0.1(0.5) + 0.15(0.5) = 1/8$. We are given that $P(F) = 1/2$, $P(F') = 1/2$, $P(E|F) = 1/10$ and $P(E|F') = 3/20$. Plugging all this data into Bayes' formula, we get

$$P(F|E) = \frac{(1/10)(1/2)}{(1/10)(1/2) + (3/20)(1/2)} = \frac{2}{5}.$$

15. (i) If we let E be the event that the first ball is yellow, and G the event that the second ball is yellow, then, by the multiplication rule, $P(E \cap G) = P(G|E)P(E) = (5/9)(4/8) = 20/72 = 5/18$. (ii) With E and G as before, we want to calculate $P(E \cup G) = P(E) + P(G) - P(E \cap G) = 5/9 + 5/9 - 5/18 = 5/6$.