

Department of Mathematics
University of Notre Dame
Math 10260 – Bus. Calc. II
Fall 2009

Name: _____

Instructor: _____

Exam I

September 17, 2009

This exam is in 2 parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in or tolerate academic dishonesty.

Signature: _____

You must record here your answers to the multiple choice problems.

Place an \times through your answer to each problem.

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
| 2. | (a) | (b) | (c) | (d) | (e) |
| 3. | (a) | (b) | (c) | (d) | (e) |
| 4. | (a) | (b) | (c) | (d) | (e) |
| 5. | (a) | (b) | (c) | (d) | (e) |
| 6. | (a) | (b) | (c) | (d) | (e) |
| 7. | (a) | (b) | (c) | (d) | (e) |
| 8. | (a) | (b) | (c) | (d) | (e) |
| 9. | (a) | (b) | (c) | (d) | (e) |
| 10. | (a) | (b) | (c) | (d) | (e) |

MC. _____

11. _____

12. _____

13. _____

14. _____

15. _____

Tot. _____

Multiple Choice

1. (5 pts.) Let $F(x)$ be a function. The Fundamental Theorem of Calculus says that the change in F from $x = a$ to $x = b$ is equal to

(a) $\int_a^b F^2(x)dx$

(b) $\int_a^b F(x)dx$

(c) the area under the graph of $F(x)$ from $x = a$ to $x = b$

(d) $\int_a^b F'(x)dx$

(e) the area under the graph of $F'(x)$ from $x = a$ to $x = b$

2. (5 pts.) Assume that you are 30 years old and you estimate that to retire comfortably at the age of 65 you will need \$1,800,600 in savings. If you can earn 9% interest compounded continuously, how much should you deposit each year in order to meet this goal? Treat the deposits as continuous income stream.

(a) $S = \frac{1,800,000}{\int_0^{35} e^{0.09(35-t)} dt}$

(b) $S = \int_0^{35} e^{0.09(35-t)} dt$

(c) $S = 1,800,000$

(d) $S = 25,000t$

(e) $S = \int_{30}^{65} e^{-0.09t} dt$

3. (5 pts.) Given the initial value problem $\frac{dy}{dx} = y^2x + y^2$ and $y(0) = 2$, find $y(3)$.

- (a) 2 (b) -6 (c) 1/6 (d) 0 (e) -1/7

4. (5 pts.) A certain population is represented by the logistic equation $\frac{dp}{dt} = .05p - .002p^2$ and initial condition $p(0) = 5$. Find the environmental carrying capacity.

- (a) 17 (b) 25 (c) 13 (d) 10 (e) 11

5. (5 pts.) Assume that you own a property which brings in a perpetual income stream flowing continuously at a rate of \$17,000 per year. What is its fair sale price assuming that money can be invested earning interest at the annual rate of 8.5% compounded continuously?

- (a) \$1,700,000
- (b) \$2,000,000
- (c) \$170,000
- (d) \$200,000
- (e) None of the above.

6. (5 pts.) Evaluate

$$\int_{-1}^0 \frac{1}{\sqrt{1-2x}} dx.$$

- (a) $\sqrt{7} - 1$
- (b) $\sqrt{2} + 1$
- (c) $\sqrt{3} - 1$
- (d) $\sqrt{11} - 1$
- (e) $\sqrt{5} + 1$

7. (5 pts.) Evaluate

$$\int x \ln 3x \, dx$$

- (a) $\frac{x^2}{2}(\ln 3x - \frac{1}{2}) + C$
- (b) $\frac{x^3}{3}(\ln 3x - \frac{1}{3}) + C$
- (c) $\frac{x^3}{2}(\ln 3x - \frac{1}{3}) + C$
- (d) $\frac{x^2}{2}(\ln 3x - \frac{1}{3}) + C$
- (e) $\frac{x^2}{3}(\ln 3x - \frac{1}{3}) + C$

8. (5 pts.) Suppose the supply and demand functions are given by $S(q) = 5 + 2q^2$ and $D(q) = 35 - 4q$. The equilibrium price is

- (a) 15
- (b) 7
- (c) -30
- (d) 23
- (e) none of the above

9. (5 pts.) Find the area between the curves $y = 1 - x^2$ and $y = x^2 - 1$ from $x = -1$ to $x = 1$.

- (a) $\frac{7}{3}$
- (b) $\frac{11}{3}$
- (c) $\frac{8}{3}$
- (d) $\frac{7}{2}$
- (e) $\frac{5}{2}$

10. (5 pts.) Use 4 subintervals and their left end points to estimate

$$\int_0^2 (-x^2 + 2x) dx$$

- (a) $\frac{3}{4}$
- (b) $\frac{7}{3}$
- (c) $\frac{5}{3}$
- (d) $\frac{7}{2}$
- (e) $\frac{5}{4}$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) (**Show your work!**) You deposit money continuously into an account at a rate of $1,000e^{0.1t}$ dollars per year, where t is time in years measured from when you open the account. If the account yields 5% interest, compounded continuously, how much money will be in the account after 10 years?

12. (10 pts.) (**Show your work!**) Evaluate

$$\int \frac{dx}{x^2 + x - 6}$$

13. (10 pts.) (**Show your work!**) Suppose the supply and demand functions are given by $S(q) = 1.3 + .06q^2$ and $D(q) = 4.3 - .3q$. Then the equilibrium quantity is 5. Find the equilibrium price and the consumer surplus.

14. (10 pts.) (**Show your work!**) Suppose the marginal cost in producing x units is given by

$$MC(x) = 2x + 17.$$

Determine the increase in cost if production is increased from 10 to 20 units.

15. (10 pts.) (**Show your work!**) The population of a city was 1 million in 2009. Assume that the population follows a logistic growth model with an intrinsic growth rate of 0.03 and an environmental carrying capacity of 5 million people.

- (i) Write an initial value problem to describe the growth of the population in this city.
- (ii) Estimate the population of that city in 2019.