Midterm 1 Answers - Bus. Calc. II
September 17, 2009

1. The answer is (d).

2. We need: $1,800,600 = \int_0^{35} Se^{.09(35-t)}dt$, where S is the rate of deposits in dollars per year. Hence the answer is (a).

3. Note $y' = dy/dt$. Separate variables and integrate to get: $\int(1/y^2)dy = \int(x+1)dx$. Hence $-y^{-1} = x^2/2 + x + c$, for some constant c. So, $y(x) = 1/(-x^2/2 - x - c)$. Plugging in $x = 0$ we get $c = -1/2$. Therefore $y(3) = 1/(-9/2 - 3 + 1/2) = -1/7$.

4. The logistic growth equation is $dp/dt = rp(1-p/K)$. We have $dp/dt = .05p(1-.002p/.05)$. Hence $K = .05/.002 = 25$.

5. The fair sale price is $\int_0^{\infty} 17,000e^{-.085t}dt = \lim_{T\to\infty}\int_0^T 17,000e^{-.085t}dt$. The integral is $17,000(e^{-.085T} - 1)/(-.085)$, and the limit is $17,000/0.085 = \$200,000$.

6. Use the substitution rule. Let $u = 1 - 2x$. Then $u'(x) = -2$. Since $u(-1) = 3$, and $u(0) = 1$, the substitution rule implies $\int_{-1}^{1} \frac{1}{\sqrt{1-2x}}\,dx = \frac{1}{-2}\int_3^1 \frac{1}{\sqrt{u}}\,du = \frac{1}{-2}2u^{\frac{1}{2}}|_3^1 = -[1-\sqrt{3}] = \sqrt{3} - 1$

7. Use integration by parts. Let $u = \ln 3x$ and $dv = xdx$, to get $du = \frac{1}{x}\,dx$ and $v = \frac{1}{2}x^2$. Then $\int x\ln 3x\,dx = uv - \int v du = \frac{1}{2}x^2\ln 3x - \frac{1}{2}\int x\,dx = \frac{1}{2}x^2[\ln 3x - \frac{1}{2}] + c$.

8. Set $5 + 2q_e^2 = S(q_e) = D(q_e) = 35 - 4q_e$. This implies $2q_e^2 + 4q_e - 30 = 0$. Since $q_2 > 0$ we get $q_e = 3$. Thus $p_e = D(q_e) = 35 - 4(3) = 23$.

9. Let $f(x) = 1 - x^2$ and $g(x) = x^2 - 1$. Since $f(x) \geq g(x)$ for all x in the interval $[-1, 1]$, we get that the area equals $\int_{-1}^{1} f(x) - g(x)\,dx = 2\int_{-1}^{1}(1 - x^2)dx = 2(x - \frac{1}{3}x^3)|_{-1}^{1} = \frac{8}{3}$

10. Let $f(x) = -x^2 + 2x$. Since we are dividing $[0, 2]$ into 4 intervals $\Delta x = \frac{1}{2}$. Our estimate equals $\Delta x[f(0) + f(\frac{1}{2}) + f(1) + f(\frac{3}{4})] = \frac{1}{2}[0 + \frac{3}{4} + 1 + \frac{3}{4}] = \frac{5}{4}$.

11. $FV = \int_0^{10} 1000e^{0.1t}e^{0.05(10-t)}dt = 1000e^{0.5}\int_0^{10} e^{0.05t}dt = 1000e^{0.5}\frac{e^{0.05t}}{0.05}|_0^{10}$. Then $FV = \$21,391.21$.

12. $\int \frac{1}{x^2+x-6}dx = \int \frac{1}{(x+3)(x-2)}dx = \int \frac{-1/5}{x+3}dx + \int \frac{1/5}{x-2}dx$. This is further equal to $-(1/5)\ln|x+3| + (1/5)\ln|x-2| + C$.

13. $p_e = D(q_e) = 4.3 - .3(5) = 2.8$.
$CS = \int_0^5 (4.3 - .3q)\,dq - (2.8)(5) = (4.3q - \frac{.3}{2}q^2)|_0^5 - 14 = 3.75$.

14. $C(20) - C(10) = \int_{10}^{20} MC(x)\,dx = \int_{10}^{20}(2x + 17)\,dx = (x^2 + 17x)|_{10}^{20} = (400 + 340) - (100 + 170) = 470$.

15. (i) $dp/dt = 0.03p(1 - p/5)$ and $p_0 = 1$.

(ii) Separating variables and integrating we get $\int \frac{1}{p(1-p/5)}dp = \int 0.03dt$. For the first integral we use that $\frac{1}{p(1-p/5)} = \frac{1}{p} + \frac{1/5}{1-p/5}$. Then we get that $\ln \frac{|p|}{|1-p/5|} = 0.03t + c$, for some constant c. Hence $\frac{p}{1-p/5} = Ce^{0.03t}$ for some constant C. Plugging in $t = 0$, we get $C = 5/4$. Hence $p(10) = (5/4)e^{0.03\cdot 10}(1 - p(10)/5)$. Solving for $p(10)$ we get that 1.26 million people will be the population in 2019.