Math 10260 Integration Review

▶ Integration by parts

- Let $u(x)$ and $v(x)$ be two differentiable functions. Applying product rule, we have:

$$\frac{d}{dx}(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$$

- By definition of anti-derivative:

$$u(x)v(x) = \underline{\hspace{6cm}} = \int u(x)v'(x)\,dx + \int u'(x)v(x)\,dx.$$

- Rearranging terms, we have:

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)\,dx$$

- Note $\frac{du}{dx} = u'(x) \quad \Rightarrow \quad du = \underline{\hspace{2cm}}$. Also $\frac{dv}{dx} = v'(x) \quad \Rightarrow \quad dv = \underline{\hspace{2cm}}$.

- Suppressing variable x, we get:

$$\boxed{\int u\; dv = \hspace{6cm} .} \rightarrow \textbf{Integration by Parts}$$

Example 1 Perform the following integration:

(a) $\int xe^{3x}\; dx$

(b) $\int x^3 \ln x\; dx$

(c) $\int_0^1 xe^{3x^2}\; dx$

(d) $\int \frac{2}{2x^2 - 3x + 1}\, dx$

(e) $\int x\sqrt{2x+9}\;dx$

(f) $\int \frac{\ln x}{x}\;dx$

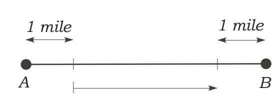
(g) $\int x\sqrt{2x^2+9}\;dx$

(h) $\int_1^2 \ln x\,dx$

Example 2 In a study of students learning foreign language, the number of new words $w(t)$ (as a function of time) an average student can learn a day is modeled by the equation $\frac{dw}{dt} = 0.1(1-t)e^{-0.1t}$ If the student begins with 20 new words a day, find w as a function of time.

Example 3 The intensity $Q(x)$ of pollution along a 5 mile long road between a site **A** in a nature reserve and a factory **B** (in still air) is given by the differential equation:

$$\frac{dQ}{dx} = \frac{20}{(x+5)(5-x)}$$



where $0 \le x < 5$ is the distance in miles measured from site **A**. Find the total change in the intensity Q of pollution experienced by a person walking along the road starting 1 mile past site **A** and ending 1 mile before factory **B** as shown in the figure below.