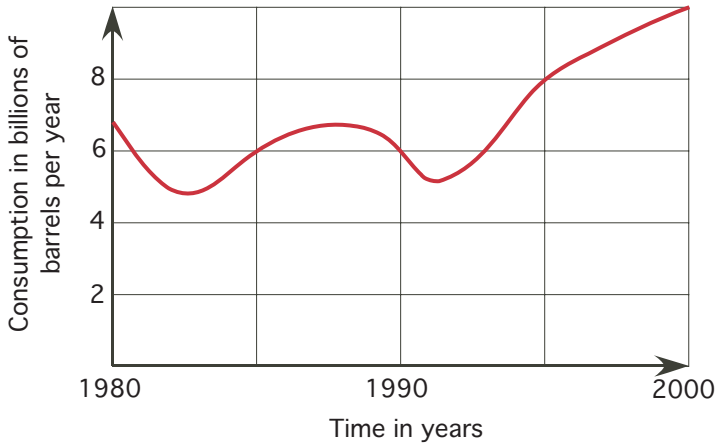
Part I: Multiple Choice Questions (5 Points Each)

1. The graph below displays the rate at which oil was consumed by the U.S. from 1980 to 2000. Use the **midpoint** rule with $\Delta t = 10$ to estimate the total amount of oil consumed during this period in billions of barrels.



(a) 70 **(b)** 100 **(c)** 140 **(d)** 200 **(e)** 180

2. Which method would you use to evaluate the integral

$$\int_1^2 \frac{\ln x}{x} dx \ ?$$

(a) integration by parts with $u = \dfrac{\ln x}{x}$ and $dv = dx$

(b) substitution with $u = \ln x$

(c) substitution with $u = \dfrac{\ln x}{x}$

(d) integration by parts with $dv = \ln x \, dx$ and $u = \frac{1}{x}$

(e) partial fractions

3. The marginal profit function of a handbag manufacturer is

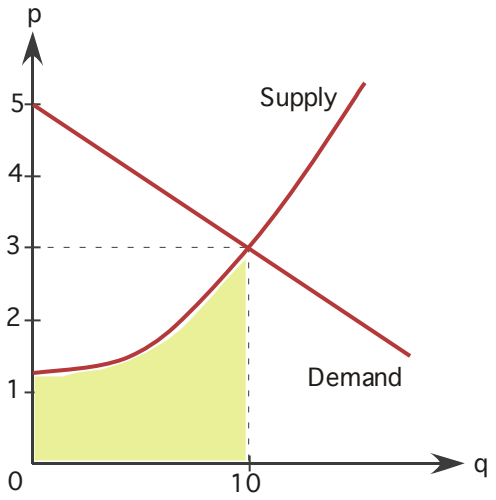
$$MP = -0.5q + 100$$

dollars per unit at the production level of q units.

Find the total change of profit from producing 10 **additional units** if 20 units are currently being produced.

(a) \$875 **(b)** \$920 **(c)** \$1,900 **(d)** \$975 **(e)** \$1,000

4. The graph below displays the demand and supply curves of a certain product. The area under the supply curve from $q = 0$ to $q = 10$ (the shaded area) is equal to 18. Then



(a) CS = 18; PS = 10 **(b)** CS = 18; PS = 12 **(c)** CS = 12; PS = 10

(d) CS = 10; PS = 12 **(e)** CS = 10; PS = 18

5. The area of the region between the curves $f(x) = x^2$ and $g(x) = 2 - x$ over the interval $[-2, 1]$ is given by which of the following integrals?

(a) $\displaystyle\int_{-2}^{0} \left[x^2 - (2 - x)\right] dx + \int_{0}^{1} \left[2 - x - x^2\right] dx$

(b) $\displaystyle\int_{-2}^{1} \left[2 - x - x^2\right] dx$

(c) $\displaystyle\int_{-2}^{0} \left[2 - x - x^2\right] dx + \int_{0}^{1} \left[x^2 - (2 - x)\right] dx$

(d) $\displaystyle\int_{-2}^{1} \left[x^2 - (2 - x)\right] dx$

(e) none of the above.

6. A company sells robots each of which generates a continuous income stream at the rate of \$84,000 per year for the buyer. Find the maximum (fair) sale price for each robot assuming that its lifespan is 8 years, and that money can be invested for that period at the annual rate of 6% compounded continuously.

(a) \$1,000,000 **(b)** \$764,972 **(c)** \$672,000

(d) \$303,000 **(e)** \$533,703

7. A home buyer plans to take a 20-year mortgage at a 4% annual interest rate, compounded continuously. Suppose that she can only afford to make a payment of \$800 **per month**. Which of the following is an initial value problem describing the amount $M(t)$ she owes at any time t between 0 and 20 years? (Assume that she will be able to pay off the loan in 20 years.)

(a) $\dfrac{dM}{dt} = 0.04M(t) - 9,600, \quad M(20) = 0$

(b) $\dfrac{dM}{dt} = 0.04M(t) + 9,600, \quad M(0) = 0$

(c) $\dfrac{dM}{dt} = 0.04M(t) - 800, \quad M(20) = 0$

(d) $\dfrac{dM}{dt} = 0.04M(t) + 800, \quad M(20) = 0$

(e) $\dfrac{dM}{dt} = 0.04M(t), \quad M(20) = 0$

8. A population of a certain region follows the logistic growth model

$$\frac{dp}{dt} = 0.02p - 0.004p^2, \quad p(0) = 2 \text{ million}.$$

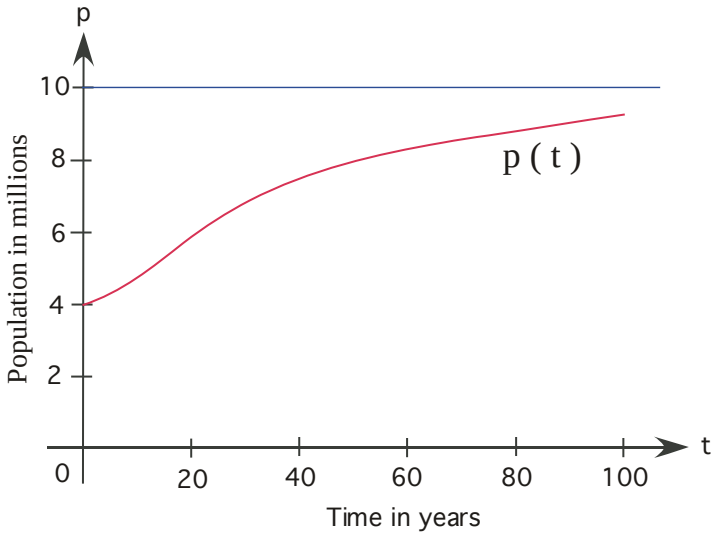
What is the environmental carrying capacity of this region?

(a) 15 million **(b)** 10 million **(c)** 3 million **(d)** 0.2 million **(e)** 5 million

9. A piece of land in the west produces a **perpetual** income stream flowing continuously at a rate of \$65,000 per year. If this income is invested at the annual rate of 6.5% compounded continuously, what is the present value of this land?

(a) \$1,200,000 **(b)** \$650,000 **(c)** \$6,500,000 **(d)** \$1,000,000 **(e)** \$500,000

10. Assume that the figure below shows the graph of the population $p(t)$, in millions, of a certain country during the last 100 years. Also, assume that this population can be modeled by a **logistic** equation. Which of the following is this equation?



(a) $\frac{dp}{dt} = 0.02p(1 - \frac{p}{5})$ **(b)** $\frac{dp}{dt} = 10p$ **(c)** $\frac{dp}{dt} = 0.03p(1 - 0.1p)$

(d) $\frac{dp}{dt} = 0.03p(1 - 10p)$ **(e)** $\frac{dp}{dt} = 0.02p(1 - \frac{p}{8})$

Part II: Partial Credit Questions (10 Points Each)

Show all work and circle your final answer, within the space provided.

No credit will be given for a correct answer without showing how it was obtained. You will receive no credit if the answer is not in the space provided and no **partial credit** for a wrong answer if you do not **show your work**.

11. Imagine that you have just started working for a great company, which offers you the following two retirement options:

    (a) It deposits money into an IRA at the variable rate of $10,000e^{0.02t}$ per year for the next 25 years.

    (b) The company pays you a lump sum of $850,000 at the end of the 25-year period.

    Assuming that the your IRA earns annual interest of 7% compounded continuously, which option will you choose? Explain your answer.

12. Suppose that for a certain item, the demand curve is

$$D(q) = \frac{49}{q+3}$$

and the supply curve is

$$S(q) = q + 3.$$

(a) Find the equilibrium quantity q_e and price p_e.

$q_e =$ _____, $p_e =$ _____

(b) Find the consumer surplus (CS).

$CS =$ _____

(c) Find the producer surplus (PS).

$PS =$ _____

13. (a) (8 points) Solve the following initial value problem

$$\frac{dy}{dx} = 3x^2(y-1), \quad y(0) = 2.$$

(b) (2 points) Check that the function $y(x) = x^4+2$ is a solution to the differential equation $xy' - 4y = -8$.

14. Imagine you have a bank account with \$1,000,000 in it. The interest rate provided is 5% annually, compounded continuously. You want to use this account for withdrawing money continuously at a steady annual rate S. What is the maximum value of S you can afford to withdraw so that your account lasts for 50 years?

15. (a) (7 points) Calculate the improper integral $\int_0^\infty xe^{-x^2}dx$.

(b) (3 points) A **perpetual** income stream is flowing into an account at the rate $S(t) = t^2e^{-0.1t} + 100$ thousands dollars per year. If the account pays interest at the annual rate of 5% compounded continuously, **write an improper integral** that gives the **present value** of this income stream. (Do **NOT** compute it.)