HW #1 - Math 10250

Section 0.2

(3) If $f(x) = 1/x$, find $f(2)$, $f(1/2)$, $f(-1)$, $f(a/3)$, $f(3/a)$, and $\frac{f(a+h)-f(a)}{h}$.

Solution: Let f be the function defined by the formula $f(x) = 1/x$, so that the natural domain consists of all $x \neq 0$.

- $f(2) = \boxed{1/2}$
- $f(1/2) = \frac{1}{1/2} \boxed{= 2}$
- $f(-1) = \frac{1}{-1} \boxed{= -1}$
- $f(a/3) = \frac{1}{a/3} \boxed{= 3/a}$
- $f(3/a) = \frac{1}{3/a} \boxed{= a/3}$
- $\frac{f(a+h)-f(a)}{h} = \frac{\frac{1}{a+h} - \frac{1}{a}}{h} = \frac{\frac{a}{a(a+h)} - \frac{(a+h)}{a(a+h)}}{h} = \frac{\left(\frac{a-a-h}{a(a+h)}\right)}{h}$ $\boxed{= \frac{-1}{a(a+h)}}$.

(10) Find the natural domain of the function $f(x) = \sqrt{1-x^2}$.

Solution: When defining the natural domain of any function, you must determine the "limits set" on that function first. For the function $f(x) = \sqrt{x}$, x cannot be negative. Therefore, in the case of $f(x) = \sqrt{1-x^2}$, the function $1-x^2$ must be greater than 0.

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(10) continued...

$$1-x^2>0$$
$$-x^2>1$$
$$x^2<1$$
$$x^2-1<0$$
$$(x-1)(x+1)<0$$
$$\boxed{-1<x<1}$$

(41) Determine whether f(x) has an inverse. If it does, find an explicit formula for the inverse and specify its domain and range. $f(x) = 1/x^2$, $x>0$.

<u>Solution</u>: First, we note that for $x>0$ the function $f(x)=1/x^2$ is 1-1. Thus it has inverse. We find it in two steps.

Step 1: solve for x as a function of y.

$$y=\frac{1}{x^2}$$
$$x^2=\frac{1}{y}$$
$$\sqrt{x^2}=\frac{1}{\sqrt{y}}$$
$$x=\frac{1}{\sqrt{y}}$$

Step 2: Next, we permute x and y to get $y=\frac{1}{\sqrt{x}}$.
Thus, the inverse function is given by

$$\boxed{g(x)=\frac{1}{\sqrt{x}},\ x>0}$$

The domain of f is $(0,\infty)$, and its range is $(0,\infty)$. Therefore, the domain of g is the set $(0,\infty)$, and its range is again $(0,\infty)$.

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Section 0.3 18. State whether the function is odd, even, or neither and what kind of symmetry, if any, its graph has.

$$f(x) = x^3 - 3x + 1$$

Solution: To determine if a function, $f(x)$, is odd [or symmetric about the origin], see if $f(-x) = -f(x)$.

$$f(x) = x^3 - 3x + 1$$
$$f(-x) = (-x)^3 - 3(-x) + 1$$
$$f(-x) = -x^3 + 3x + 1$$

So, $f(-x) \neq -f(x)$.

To determine if a function, $f(x)$, is even [or symmetric about the y-axis], see if $f(-x) = f(x)$.

$$f(-x) = (-x)^3 - 3(-x) + 1$$
$$f(-x) = -x^3 + 3x + 1$$

So, $f(-x) \neq f(x)$.

$\boxed{\text{Thus, } f \text{ is neither even nor odd.}}$

40. Apply translations to the graph of $y = x^2$ to obtain the graph of the given equation.

$$y = (x+2)^2 - 1$$

Solution: The equation can be written as

$$y = (x-h)^2 + k,$$

where h is the horizontal translation and k is the vertical translation.

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(40) continued... Therefore, the graph of $y=(x+2)^2-1$ is obtained by translating the graph of $y=x^2$ left two units and down one unit.