Math 10250, Practice B – Final Exam
December 11, 2018

- Be sure that you have all 17 pages of the test.
- Calculators are allowed for this examination.
- The exam lasts for two hours.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- **Sign the pledge.** "As a member of the Notre Dame Community, I will not participate in or tolerate academic dishonesty":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	26.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	27.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)	28.	(a)	(b)	(c)	(d)	(e)
13.	(a)	(b)	(c)	(d)	(e)	29.	(a)	(b)	(c)	(d)	(e)
14.	(a)	(b)	(c)	(d)	(e)	30.	(a)	(b)	(c)	(d)	(e)
15.	(a)	(b)	(c)	(d)	(e)						
16.	(a)	(b)	(c)	(d)	(e)						

Multiple Choice

1.(5 pts.) The relation between temperature in degrees Fahrenheit (F) and temperature in degrees Celsius (C) is

$$F = \frac{9}{5}C + 32.$$

Which of the following statements is FALSE?

(a)   The inverse relation is: $C = \frac{5}{9}(F - 32)$

(b)   When $F = 32$ then $C = 0$.

(c)   When $C = 100$ then $F = 212$.

(d) F is a quadratic function of C.

(e) F is a linear function of C.

2.(5 pts.) The value of a painting now is \$20,000 while 10 years ago it was \$10,000. Assuming that it is modeled by a linear function, find a formula for its value at any time t in the future.

(a) $V(t) = 1000t + 20,000$

(b) $V(t) = 100t + 20,000$

(c) $V(t) = t + 20,000$

(d) $V(t) = -1000t + 20,000$

(e) $V(t) = 10t + 20,000$

3.(5 pts.) Martin's bakery estimates that the demand curve for its Mediterraneo bread is $p = -q + 6$, while the supply curve is $p = q + 2$, where q is measured in hundreds of loaves and the price p is in dollars. Which of the following statements is FALSE?

(a) The equilibrium price is $p = 4$.

(b) The equilibrium quantity is $q = 2$.

(c) The supply function is increasing.

(d) The demand function is decreasing.

(e) The slope of the demand curve is equal to 6.

4.(5 pts.) When x units of an item are produced then its market price (in dollars) is $p = -0.5x + 100$. Also, the cost per unit is 20 dollars and the fixed costs are 500 dollars. Which of the following statements is FALSE?

(a) The cost function is $C(x) = 20x + 500$.

(b) The marginal profit is $MP(x) = -x + 80$.

(c) The revenue function is $R(x) = -0.5x^2 + 100x$.

(d) The marginal cost is $MC(x) = 500$.

(e) The profit function is $P(x) = -0.5x^2 + 80x - 500$.

5.(5 pts.) During the last 90 years, the average return on a risk free government bond is 0.7% after adjusting for inflation, while the average return of stocks is about 8%, again after adjusting for inflation. Assume that this trend continues during the next 40 years and that today you invest \$100 in this government bond and \$100 in stocks. Which of the following statements is FALSE, if in both cases interest is computed continuously.

(a)   After 40 years, the future value of your \$100 in stocks will be $100e^{3.2}$.

(b)   After 40 years, the future value of your \$100 in bonds will be $100e^{-0.28}$.

(c)   At any time $t \le 40$, the future value of your \$100 in stocks will be $100e^{0.08t}$.

(d)   After 40 years, your \$100 in stocks will grow to $e^{2.92}$ times of your \$100 in bonds.

(e)   At any time $t \le 40$, the future value of your \$100 in bonds will be $100e^{0.007t}$.

6.(5 pts.) Assuming an annual stock return of 8% compounded contunuously, find how much you should invest in stocks now so that in 40 years you have \$2,000,000.

(a) $2{,}000{,}000\, e^{-3.2} \approx 81{,}524$

(b)   40,000

(c)   50,000

(d)   80,000

(e)   20,000

7.(5 pts.) From now on ($t \geq 0$), assume that the GDP (gross domestic product) of a country A is evolving according to the formula $G_A(t) = 10e^{0.03t}$, while the GDP of a country B is evolving according to the formula $G_B(t) = 5e^{0.08t}$. Both GDP are measured in trillions of dollars and the time t in years. Which of the following statements is FALSE?

(a)   Both countries will have the same GDP at $t = 20 \ln 2$.

(b)   At $t = 0$ we have $G'_A(0) = 0.3$ and $G'_B(0) = 0.4$.

(c)   Country A will have always bigger GDP.

(d)   Now the GDP of country B is 5 trillion dollars.

(e)   Now the GDP of country A is 10 trillion dollars.
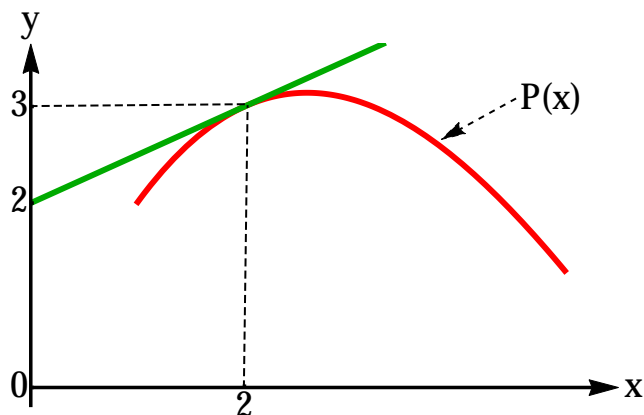
8.(5 pts.) For any given $x > 0$ find the limit:
$$\lim_{h \to 0} \frac{\ln(x + h) - \ln x}{h}.$$
(*Hint: Think Derivative!*)

(a) $\ln x$

(b) x

(c)   None of these.

(d) $\dfrac{1}{x}$

(e) e^x

9.(5 pts.) The profit function $P(x)$ from the production and selling of x thousands of units of an item is displayed in the figure below. Find the marginal profit at the production level of $x = 2$.

(a)   2/3

(b)   1/2

(c) $-1/2$

(d)   3/2

(e)   None of these.

10.(5 pts.) The profit function $P(x)$ from the production and selling of x thousands of units of an item is displayed in the figure above. Find its linear approximation at the production level of $x = 2$.

(a) $P(x) \approx 3 + 0.5(x - 2)$

(b) $P(x) \approx 3 + 0.5x$

(c)   None of these.

(d) $P(x) \approx 3 - 0.5(x - 2)$

(e) $P(x) \approx 0.5(x - 2)$

11.(5 pts.) Let $f(x)$ be the function, whose graph is shown below, and $g(x)$ be a differentiable function at $x = 3$ with $g(3) = 3$ and $g'(3) = 9$. Find the instantaneous rate of change of the function $f(g(x))$ at $x = 3$.

(a)   None of these.

(b) -10

(c) -12

(d)   10

(e)   12
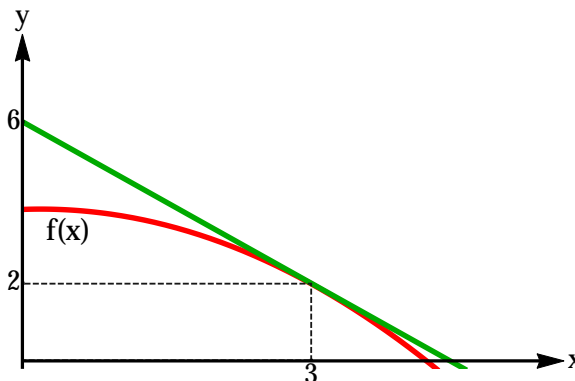
12.(5 pts.) In an economy, the capital per worker k and its output per worker q are related by the formula
$$q = 800k^{1/2}.$$
Currently $k = 10{,}000$ dollars and it is changing at the rate of 1,000 dollars per year. Find the rate at which the output is changing.

(a)   None of these.

(b)   2,000

(c)   80,000

(d)   8,000

(e)   4,000

13.(5 pts.) In 2014, GM had \$155 billion in revenue. If the marginal revenue in 2014 was \$2.5 billion, then use linear approximation to estimate the revenue for 2018.

(a)   155

(b)   160

(c)   165

(d)   145

(e)   150

14.(5 pts.) In economics, a utility function u assigns $u(x)$ units of satisfaction (utiles) to x units of consumption. It is required to satisfy the conditions:

  i) $u'(x) > 0$ (the more the consumption the more the satisfaction)

  ii) $u''(x) < 0$ (each additional unit of consumption gives less satisfaction)

  Which one of the following functions is NOT a utility function?

(a) $u(x) = 1 - e^{-x}$

(b) $u(x) = x^2$

(c) $u(x) = x^{2/3}$

(d) $u(x) = \ln x$

(e) $u(x) = \sqrt{x}$

15. (5 pts.) The position of an object moving on a straight line is given by
$$s(t) = 2(t - 1)^3 + 1.$$
Find on what time interval the object is accelerating and on what time interval the object is decelerating.

(a)   Accelerating for $t > 1$ and decelerating for $t < 1$.

(b)   Decelerating for $t > 1$ and accelerating for $t < 1$.

(c)   Accelerating for all time.

(d)   Deceleratingfor all time.

(e)   Accelerating: $t > 12$, Decelerating: $t < 12$.

16. (5 pts.) The profit P (in millions of dollars) of a company from selling x millions of units of its product is given by
$$P(x) = 5x^2 e^{-x}, \quad 0 \le x < \infty.$$
Find the quantity x, which maximizes the profit.

(a) $x = 1$

(b) $x = 5$

(c) $x = 10$

(d)   None of these.

(e) $x = 2$

17.(5 pts.) The demand for an item is given by $p = \dfrac{400}{x+2}$, where p is the price and x is the quantity (in millions). Find the maximum revenue that can be obtained assuming that the item is available in any quantity demanded ($x \geq 0$).
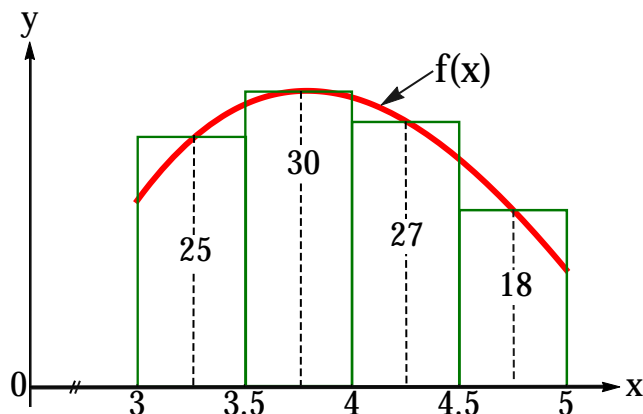
(a)   200

(b)   There is no maximum revenue.

(c) $\dfrac{400}{(x+2)^2}$

(d)   400

(e)   800

18.(5 pts.) In problem 17, find the maximum profit assuming the cost for each unit of the item is $8.

(a)   320

(b)   200

(c)   400

(d)   There is no maximum profit.

(e)   256

19.(5 pts.) For the function $f(x)$, whose graph is displayed in the figure below, which of the following statements is TRUE.

(a) $\int_3^5 f(x)dx = 0$.

(b) $\int_3^5 f(x)dx = 60$.

(c) $\int_3^5 f(x)dx \approx 50$.

(d) $\int_3^5 f(x)dx = 80$.

(e) $\int_3^5 f(x)dx = 36$.

20.(5 pts.) The instantaneous rate of change of a quantity $Q(t)$ is given by the formula:
$$Q'(t) = 4t\ln t, \ 1 \le t \le 5.$$
Compute the total change of this quantity when t changes from 1 to 5.

(a) $50\ln 5$.

(b) $4\ln 5 - 24$.

(c)   None of these.

(d) $50\ln 5 - 24$.

(e)   24.

21.(5 pts.)The Figure below displays the rate $r(t)$ (in billion of gallons per year) at which gas was consumed in a certain region during the last 20 years. Find the total amount of gas (in billions of gallons) consumed in this period.

(a)   None of these.

(b)   175

(c)   200

(d)   150

(e)   100
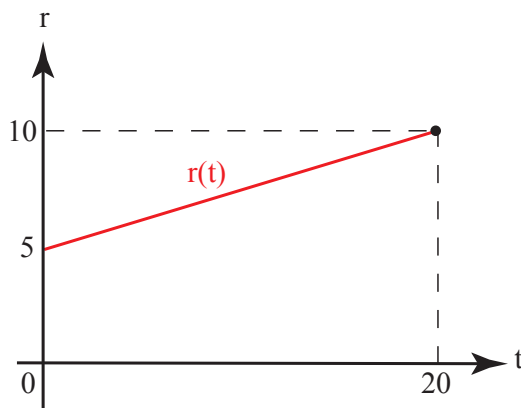
22.(5 pts.) It is projected that in a region, during the next 10 years, oil will be consumed at the rate
$$r(t) = 30 + 2te^{-0.01t^2}$$
in millions of barrels per year. Find the total oil consumption projected for this period.

(a) $400 - 100e^{-1}$

(b) $300e^{-1}$

(c) 400

(d) $400 + 100e^{-1}$

(e) $100e^{-1}$

23.(5 pts.) The GDP (Gross Domestic Product) $y(t)$ of a certain county is modeled by the differential equation
$$\frac{dy}{dt} = 0.02y,$$
where y is measured in trillions of dollars and the time t in years. If currently the GDP of this country is \$4 trillion, find its GDP after t years.

(a) $y(t) = 0.2e^{0.02t}$

(b) $y(t) = e^{0.02t}$

(c) $y(t) = e^{0.02t} + 3$

(d)   None of these.

(e) $y(t) = 4e^{0.02t}$

24.(5 pts.) A function $f(x)$ on an interval $[a, b]$ is a probability density function (pdf) if $f(x) \geq 0$ and $\int_a^b f(x)dx = 1$. Find c such that $f(x) = c(9 - x^2)$ is a pdf on $[0, 3]$.

(a)   None of these.

(b)   3/18

(c)   1/18

(d)   4/18

(e)   2/18

25.(5 pts.) The figure below displays the marginal profit $MP(x)$ for an item at the production level x, where the quantity x is measured in millions of units. Which of the following is the **best** estimate for the total change in profit (in millions of dollars) when the production level x changes from 0 to 2.5 millions of units.

(a)　8.5

(b)　10.5

(c)　16

(d)　9.5

(e)　7.5
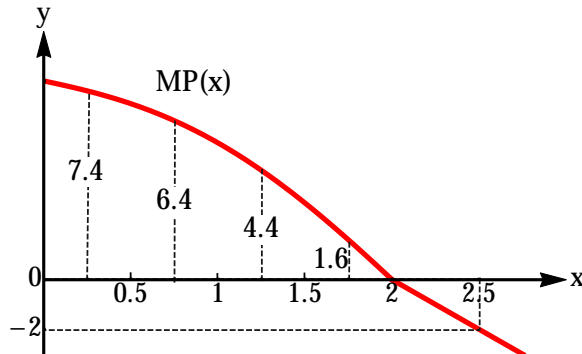
26.(5 pts.) A company has determined that when it produces at least 50 units of its product, then its marginal cost is modeled by $MC(x) = 0.1x + 80$ and its marginal revenue by $MR(x) = -0.1x + 110$. If this company is currently operating at a production level of 50 units per day, find the change in profit if the company increases production to 100 units per day?

(a)　750

(b)　550

(c)　950

(d)　650

(e)　850

27.(5 pts.) Suppose the half-life of a radioactive substance is 10 years. How long will it take for the substance to be reduced to 20% of its initial amount.

(a)   None of these.

(b) $t = -\dfrac{\ln(0.2)}{0.1 \ln 2}$

(c) $t = \dfrac{\ln 2}{0.1 \ln 2}$

(d) $t = \dfrac{\ln(0.2)}{0.1 \ln 2}$

(e) $t = 10 \ln 2$

28.(5 pts.) For the function defined by the formula $f(x) = \dfrac{e^{2x} - 1}{e^{2x} + 1}$, which of the following statements is FALSE.

(a) $f(0) = 0$ and $f'(0) = 1$

(b) $y = 1$ is a horizontal asymptote.

(c)   There is a point x where the derivative of $f(x)$ is zero (critical point).

(d) $f(x)$ is increasing for all x.

(e)   The linear approximation at $x = 0$ of $f(x)$ is: $f(x) \approx x$

29.(5 pts.) A paint manufacturer needs to construct a cylindrical can that holds 2000π cm^3 of its product. To reduce its cost for the can, the manufacturer needs to construct one with minimal surface area. Find the dimensions of such a can.

(a) $r = 10,\ h = 10$

(b) $r = 5,\ h = 10$

(c) $r = 20,\ h = 20$

(d) $r = 5,\ h = 5$

(e) $r = 10,\ h = 20$

30.(5 pts.) Mark the statement that is FALSE.

(a) $\ln e = 1,\ \ \ln 1 = 0$ and $e^{\ln 2} = 2$.

(b)   Isaac Newton is famous **not** for his calculus ideas but for his delicious fig newtons.

(c)   The Future and Present Values are related by the formula $FV = e^{rT}PV$.

(d)   If $F(x)$ is an antiderivative of a continuous function $f(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$.

(e)   A cylinder of radius r and height h has surface area of $2\pi rh + 2\pi r^2$ and volume $\pi r^2 h$.

Name: _____

Math 10250, Practice B – Final Exam   Instructor: __ANSWERS__
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