Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc. I

Name:_____

Instructor:_____

Practice B – Exam 3

November 14, 2018

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

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Place an × through your answer to each problem.

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Multiple Choice

1. (5 pts.) Suppose that a culture of Ebola viruses in a special laboratory in NYC Mount Sinai Hospital replicates satisfying the **initial value problem** $P'(t) = 0.02P(t)$, $P(0) = 100$, where t is measured in minutes. Find a formula $P(t)$ for the number of Ebola viruses in the culture after t minutes.

(a) $P(t) = e^{0.02t}$

(b) $P(t) = 100$

(c) $P(t) = te^{0.02t}$

(d) $P(t) = 10e^{2t}$.

(e) $P(t) = 100e^{0.02t}$

2. (5 pts.) The marginal profit function of a company producing and selling x personal computers per day is $MP(x) = -x + 1000$. Since $P(0) = 0$ (in other words the profit from selling 0 computers is \$0), what is the profit from selling 10 personal computers?

(a) \$10000

(b) \$9950

(c) \$5000

(d) \$7500

(e) \$10050

3. (5 pts.) The North Dakota oil boom started in 2006 with the discovery of Parshall Oil Field. Assume that the oil production in North Dakota (in billion of barrels per year) is modeled by the function

$$P(t) = 30 + te^{-\frac{1}{50}t},$$

where t is measured in years (after 2006). Find the year at which oil production will peak.

(a) 2006

(b) 2050

(c) 2056

(d) Oil production will increase forever.

(e) 2075

4. (5 pts.) The second derivative of the differentiable function $y(x)$ is given by

$$y''(x) = (x^2 - 1)^2(2x - 3)^4.$$

Which of the following is **TRUE**.

(a) There are no inflection points.

(b) The inflection points are $x = 1$, $x = -1$ and $x = \frac{3}{2}$

(c) The only inflection points is $x = \frac{3}{2}$

(d) The inflection points are $x = 1$ and $x = -1$

(e) The inflection points are $x = 0$ and $x = 1$

5. (5 pts.) Find all critical points of the function $f(x) = x^3 - 4x^2 + 5x - 2$.

(a) The only critical point is $x = \frac{5}{3}$.

(b) The critical points are $x = -1$ and $x = -3$.

(c) The critical points are $x = 1$ and $x = \frac{5}{3}$.

(d) f(x) does not have any critical point.

(e) The only critical point is $x = 1$.

6. (5 pts.) For a function $f(x)$, it is given to you that $x = 0$ is a **critical point** and that its **second** derivative is $f''(x) = e^{-x}(x^2 - 4x + 2)$. Which of the following statements is **FALSE** for this function?

(a) The graph of $f(x)$ is concave down when $x < 2 - \sqrt{2}$.

(b) $f'(x) = 0$ for $x = 0$.

(c) $f(x)$ changes concavity at $x = 2 + \sqrt{2}$.

(d) The graph of $f(x)$ is concave up when $x > 2 + \sqrt{2}$.

(e) $f(x)$ does not change concavity at $x = 0$.

7. (5 pts.) The "Mademoiselle Sucré" bakery in Paris is currently selling x croissants per day and its revenue function is given by $R(x) = 50 - 50e^{-x}$. Find the number (if any) of croissants to be sold in order to maximize and minimize the revenue of the bakery.

(a) Max for $x = 50$, Min does not exist.

(b) Max for $x = 50$, Min for $x = 5$.

(c) Max does not exist, Min does not exist.

(d) Min for $x = 0$, Max does not exist.

(e) Max for $x = 50$, Min for $x = 0$.

8. (5 pts.) Let $y(x) = \frac{x}{x-1}$. Determine which of the following statements is **FALSE.**

(a) $y(x)$ is decreasing.

(b) $y(x)$ has a vertical asymptote at $x = 1$.

(c) The graph of $y(x)$ is concave up.

(d) $y(x)$ has a horizontal asymptote at $y = 1$.

(e) The graph of $y(x)$ is concave up for $x > 1$.

9. (5 pts.) By using implicit differentiation, find the slope of the tangent line to the circle of equation $x^2 + y^2 = 4$ at the point $(\sqrt{2}, \sqrt{2})$.

(a) $-\sqrt{2}$

(b) The slope is not defined at that point.

(c) $\sqrt{2}$

(d) 1

(e) -1

10. (5 pts.) Which of the following statements is **FALSE** for a function $f(x)$ whose **derivative** is given as in the graph below? (Note that the graph shown is for $f'(x)$, **NOT** for $f(x)$.)

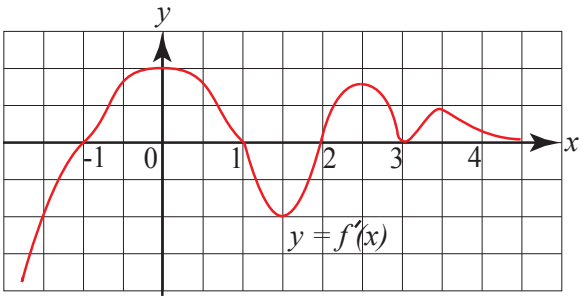
(a) $x = -1$, $x = 1$ and $x = 2$ are the only critical points of $f(x)$.

(b) $f(x)$ has a local minimum at $x = -1$.

(c) $f(x)$ is increasing for $x > 3$.

(d) $f(x)$ is decreasing for $x < -1$.

(e) $f(x)$ has a local maximum at $x = 1$.



Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) **Show your work. Circle your answers.**

(i) (6 points) Suppose that the Marginal Revenue function of Ford (in tens of thousands of dollars) for selling x of its new aluminum F-150 pick up is given by $MR(x) = x^5 - e^{-3x} + x + 1$. Assuming that $R(0) = \$0$, find the revenue function $R(x)$.

(ii) [Independent of (i)] (6 points) Compute $\displaystyle\int \frac{3x^5 + x}{x^6 + x^2 + 12}\, dx$.

12. (12 pts.) **Show your work. Circle your answers.**

(i)(6 points) Find all the critical points of the polynomial $y(x) = 2x^3 - 9x^2 + 12x + 6$.

(ii) [Independent of (i)] (6 points) The polynomial $p(x) = 2x^3 - 3x^2 + 10$ has only two critical points at $x = 0$ and $x = 1$. Find the absolute maximum and minimum points for $p(x)$ on the **closed** interval $[-1, 2]$.

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13. (13 pts.) A manufacturer has to construct a cylinder with volume 20π cubic feet. The material for the top and the base of the cylinder costs \$2 per square feet. The material for the side costs \$3 per square feet. Find the best dimensions in order to minimize the cost of the materials. Please complete the following steps.

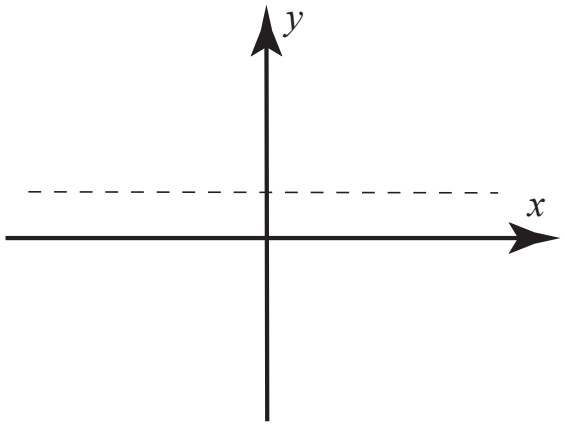
- Draw the cylinder and label the relevant variables.
- Find the cost function as a function of the radius only.
- Minimize the cost function and then find the optimal radius and height.

14. (13 pts.) Sketch the graph of the function

$$f(x) = \frac{e^x}{e^x - 1}$$

by completing the following steps.

- Find all x-intercepts and y-intercept.
- Find all the critical points, vertical and horizontal asymptotes.
- Determine where $f(x)$ is increasing and where it is decreasing.
- Study the concavity of $f(x)$.



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