

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

## Practice B – Exam 2

October 8, 2014

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

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MC. \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

14. \_\_\_\_\_

Tot. \_\_\_\_\_

**Multiple Choice****1.** (5 pts.)

A colony of fruit flies had a population of 5,000 when first observed, and 5 days later it had grown to 10,000. Assuming the population is growing exponentially, find the population 15 days after the initial observation.

- (a) 20,000
- (b) 80,000
- (c) 15,000
- (d) None of these
- (e) 40,000

**2.** (5 pts.) Evaluate the limit

$$\lim_{h \rightarrow 0} \frac{\ln(5+h) - \ln 5}{h}.$$

- (a) Does not exist
- (b) 1/5
- (c) 1/ln 5
- (d) 5
- (e) ln 5

3. (5 pts.) Find the tangent line to  $f(x) = e^{x^2}$  at  $x = -1$ .

(a)  $y = x^2 + e$

(b)  $2ex + e$

(c)  $y = -2ex - e$

(d) Does not exist

(e)  $y = -2ex + 2e$

4. (5 pts.) A baker makes 200 donuts per day. At that production level, his profit on donuts is \$100, the marginal revenue is \$0.15 per donut and the marginal cost is \$0.05 per donut. Using linear approximation, estimate how much the baker's profit for making donuts will be if he increases production by 20 donuts a day.

(a) \$104

(b) \$ 98

(c) \$ 101

(d) \$102

(e) \$ 101.5

5. (5 pts.) Find the second derivative of  $y = \ln(x^3)$

(a)  $y'' = 3 \ln(x^2)$

(b)  $y'' = 3x \ln(x)$

(c)  $y'' = -3x^{-2}$

(d)  $y'' = 6x$

(e)  $y'' = x^{-3}$

6. (5 pts.) Solve the following equation:  $3 \ln(2x) - \ln(8x) = 2$ .

(a)  $x = 1$

(b)  $x = -1$

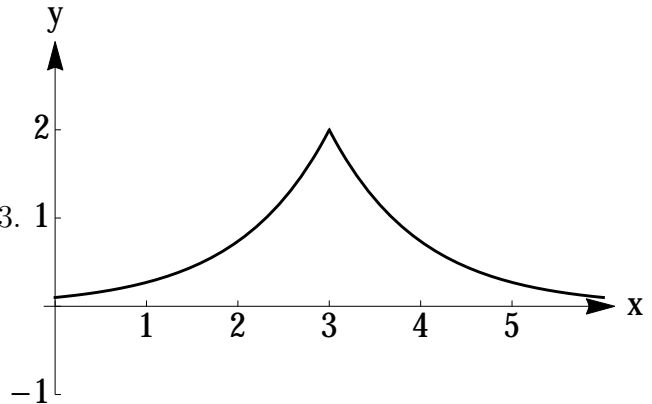
(c)  $x = 0$

(d)  $x = e$

(e) there is no solution

7. (5 pts.) Which of the following statement concerning the graph of  $f(x)$  on the right is FALSE?

- (a)  $f(x)$  is differentiable everywhere.
- (b)  $f(x)$  is differentiable except  $x = 3$ .
- (c) The graph of  $f(x)$  is symmetric around  $x = 3$ .
- (d)  $f(x)$  has positive derivative on  $(0, 3)$ .
- (e)  $f(x)$  is continuous everywhere.

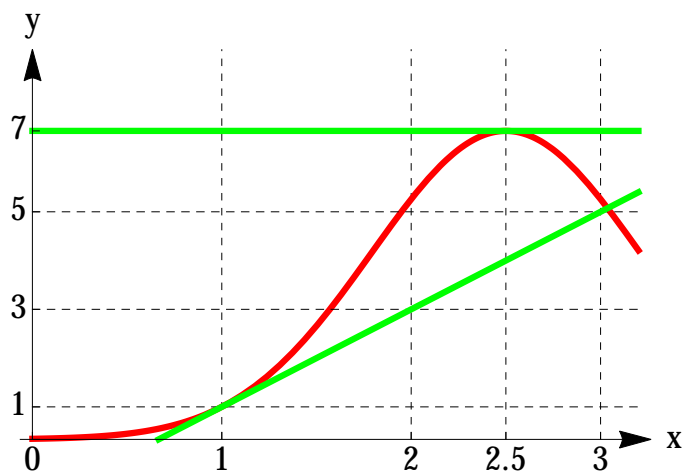


8. (5 pts.) A farmer producing the world famous Pont L'Évêque cheese has a fixed cost of \$1,000 per week, plus a unit cost of \$5 per week for each block of cheese he produces. The demand function is given by  $p = 20 - 0.01x$ , where  $x$  is the number of blocks produced each week. If every block of cheese is sold, find the marginal profit when  $x = 100$ .

- (a)  $MP(100) = 0$
- (b)  $MP(100) = 19$
- (c)  $MP(100) = 13$
- (d)  $MP(100) = 400$
- (e)  $MP(100) = 750$

9. (5 pts.) Given the graph of  $f(x)$  below, find the derivative of the function  $q(x) = \frac{f(x)}{x+1}$  at  $x = 1$ .

- (a)  $1/2$   
 (b)  $0$   
 (c)  $1$   
 (d)  $4/3$   
 (e)  $3/4$



10. (5 pts.) With  $f$  defined by the same graph as above, let  $g(x) = f(2x^2 + 0.5)$ . Find  $g'(1)$ .
- (a)  $8$   
 (b)  $0$   
 (c)  $4$   
 (d) does not exist  
 (e)  $2$

**Partial Credit**

You must **show your work** on the partial credit problems to receive credit!

**11.** (13 pts.) [**Show your work**][**Show your work!**]

(A) Iodine-131 is a radioactive substance used to treat thyroid disorders. Assume Iodine-131 decays exponentially. Suppose that 25% of an initial amount of Iodine-131 decays after 3 days (so 75% REMAINS at this time).

(i) (5 points) Find the formula for the amount of Iodine-131 which remains after  $t$  days.

**Answer:** \_\_\_\_\_

(ii) (4 points) What proportion of the Iodine-131 will remain after 2 days?

**Answer:** \_\_\_\_\_

(B) (Independent of A.) (4 pts.) If \$ 10,000 is deposited into an account paying 5% annual interest compounded continuously, exactly how long does it take before there is \$30,000 in the account?

**Answer:** \_\_\_\_\_

**Partial Credit**

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**12.** (13 pts.) [**Show your work**][**Show your work!**]

(A) A ball is thrown into the air and its height in feet (measured from the ground) after  $t$  seconds is given by  $s = -16t^2 + 48t + 64$  until it hits the ground.

(i) (4 pts.) From what height was the ball thrown? What was its initial velocity? Was it thrown upward or downward?

**Initial height:** \_\_\_\_\_

**Initial velocity:** \_\_\_\_\_

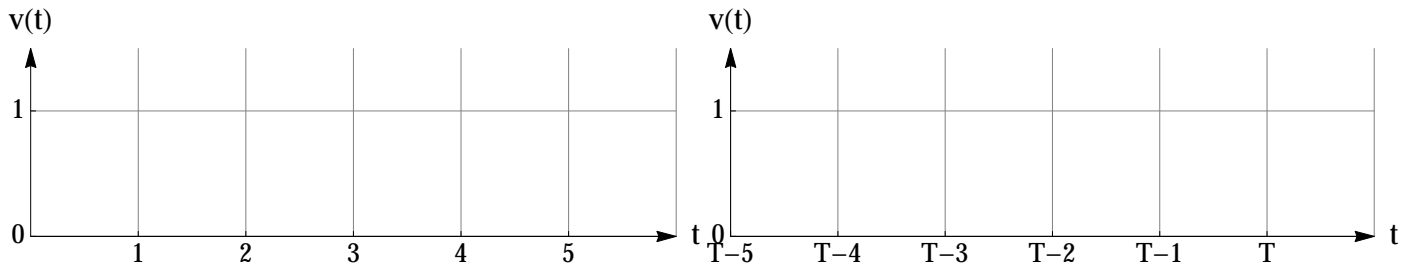
**Up or down?:** \_\_\_\_\_

(ii) (3 pts.) Find the acceleration at any time you like.

**Answer:** \_\_\_\_\_

(B) (Independent of A.) A car enters the Indiana toll road at the South Bend entrance heading east. During the first minute the car travels 1 mile accelerating to a speed of 70 miles/hour ( $= 7/6$  miles/ minute), a speed it maintains for the next 84 miles, 1 miles from the east gate, at which point it decelerates over the next minute to a complete stop at the gate.

(i) (3 points) Draw a sketch of the velocity (in miles/minute) of the car over the first and last five minutes of the trip ( $T$  is the number of minutes the trip took).



(ii) (3 points) What was the average speed of the car?

**Answer:** \_\_\_\_\_



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**13.** (12 pts.) [**Show your work**][**Show your work!**]

(A) Consider the function  $f(x) = \sqrt{x}e^{x^2}$  for  $x > 0$ .

(i) (4 points) Explain why  $f$  is differentiable and find its derivative  $f'(x)$ .

**Answer:** \_\_\_\_\_

(ii) (4 points) Use linear approximation to estimate  $f(4.1)$ .

**Answer:** \_\_\_\_\_

(B) (Independent of A.) Consider the function defined by

$$g(x) = \begin{cases} -x^3, & x \leq 0 \\ x^3, & x \geq 0 \end{cases}$$

(i) (2 points) Write down the slope of the secant line through  $(0, g(0))$  and  $(h, g(h))$

**Answer:** \_\_\_\_\_

(ii) (2 points) Determine if  $g(x)$  is differentiable at  $x = 0$ , and if so find its derivative.

**Answer:** \_\_\_\_\_

**14.** (12 pts.) [**Show your work**][**Show your work!**]

(A.) The Federal FICA tax rate is 6% for annual incomes up to \$115,000, and is a flat amount of \$6,900 for annual incomes above \$115,000.

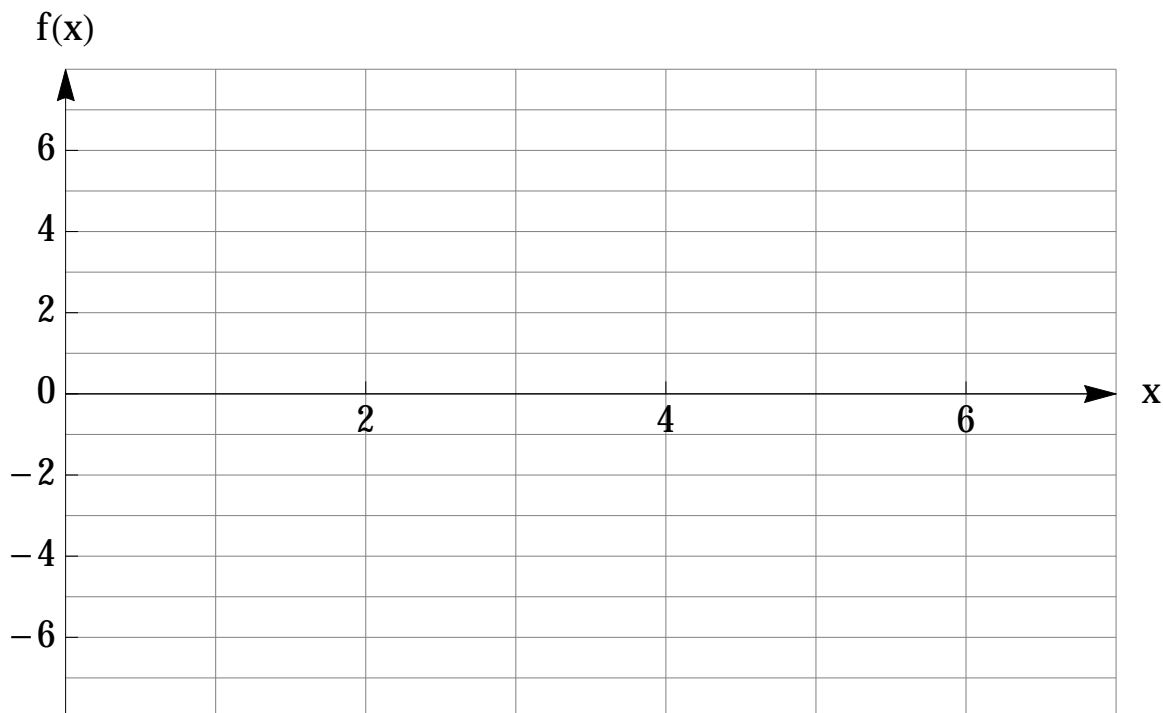
(i) (4 points) Write a formula for  $T(x)$ , the FICA tax due for an annual income of  $x$  (in thousands of dollars), or, sketch the graph of  $T$  (with all pertinent information).

**Answer:** \_\_\_\_\_

(ii) (2 points) Is  $T$  a differentiable function at every point in its domain? Explain.

(B) (Independent of A.) (6 pts.) Sketch the graph of a function  $f(x)$  defined on  $[0, 6]$  with each of the following properties:

- (a)  $f(x)$  is continuous on  $[0, 6]$
- (b) The graph of  $f(x)$  has a tangent line at all but one point (allowing vertical ones)
- (c)  $f(x)$  is differentiable at all but two points



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