Name: _____

Practice A, Math 10250, Final Exam Instructor: _____

- Be sure that you have all 16 pages of the test.
- Calculators are allowed for this examination.
- The exam lasts for .
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- **Sign the pledge.** "As a member of the Notre Dame Community, I will not participate in or tolerate academic dishonesty":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	20.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	22.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	23.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	26.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	27.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)	28.	(a)	(b)	(c)	(d)	(e)
13.	(a)	(b)	(c)	(d)	(e)	29.	(a)	(b)	(c)	(d)	(e)
14.	(a)	(b)	(c)	(d)	(e)	30.	(a)	(b)	(c)	(d)	(e)
15.	(a)	(b)	(c)	(d)	(e)						
16.	(a)	(b)	(c)	(d)	(e)						

Multiple Choice

1.(5 pts.) Mark the statement that is **false**.

(a) If $F(x)$ is an antiderivative of a continuous function $f(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$.

(b) The definite integral of $f(x)$ is: $\displaystyle\int_a^b f(x)dx = \lim_{n\to\infty}[f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x]$.

(c) The derivative of $f(x)$ is: $\displaystyle f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$.

(d) The Present Value of a continuous stream $S(t)$ is: PV$= \int_0^T S(t)e^{-rt}dt$.

(e) Isaac Newton is **only** famous for his delicious fig newtons, and **not** for his calculus ideas.

2.(5 pts.) When the price of an item is \$5 then the quantity sold is 90 units. However if the price is raised to \$7 then the quantity sold falls to 50 units. Find the demand function, assuming that it is linear.

(a) $q = -20p + 190$

(b) $q = 20p + 190$

(c) None of these

(d) $q = 20p - 190$

(e) $q = -20p - 190$

3.(5 pts.) Assume that the demand function for an item is $p = -0.1q + 40$, where price p is in dollars and the quantity q in units. If the cost per unit is \$5 then find the profit function $P(q)$.

(a) $P(q) = 0.1q^2 - 35q$

(b) $P(q) = -0.1q^2 - 40q$

(c) $P(q) = -0.1q^2 + 35q$

(d) $P(q) = -0.1q^2 - 35q$

(e) $P(q) = -0.1q^2 + 40q$

4.(5 pts.) The profit (in thousands of dollars) from selling x units of an item is given by the function

$$P(x) = \frac{50x}{x+4} - 2x.$$

Find the marginal profit when $x = 6$.

(a) 1

(b) 5

(c) 4

(d) 0

(e) ∞

5.(5 pts.) Assume that the temperature in degrees Fahrenheit of a freshly made cup of tea is modeled by the function

$$H(t) = 400e^{-0.1t},$$

where the time t is measured in minutes. Find the instantaneous rate of change of the tea temperature 10 minutes later.

(a) $-40e^{-1}$

(b) $40e$

(c) $40e^{-1}$

(d) $-40e$

(e) ∞

6.(5 pts.) Find $\lim_{h\to 0} \dfrac{\ln(5+h) - \ln 5}{h}$. (Think derivative!)

(a) 0.3

(b) 0.2

(c) 0.4

(d) ∞

(e) 0.5

7.(5 pts.) For the function $f(x)$, whose graph is given in Figure 1, compute $f'(6)$.

(a) 2

(b) 0

(c) -2

(d) 15

(e) ∞

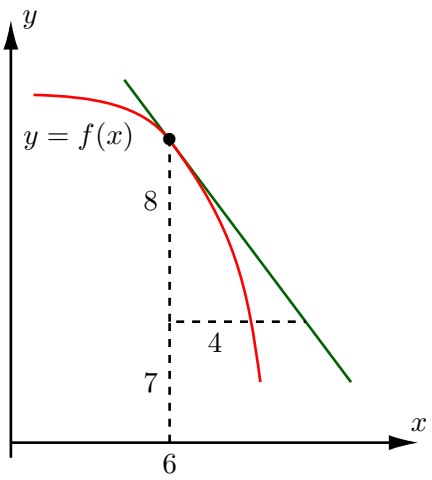

Figure 1

8.(5 pts.) For the function $f(x)$, whose graph is given in Figure 2, use linear approximation to estimate $f(1.2)$. (Notice the **different scales** in the x and y axis.)

(a) 4.4

(b) 4.8

(c) 4.1

(d) 3.6

(e) 4.3

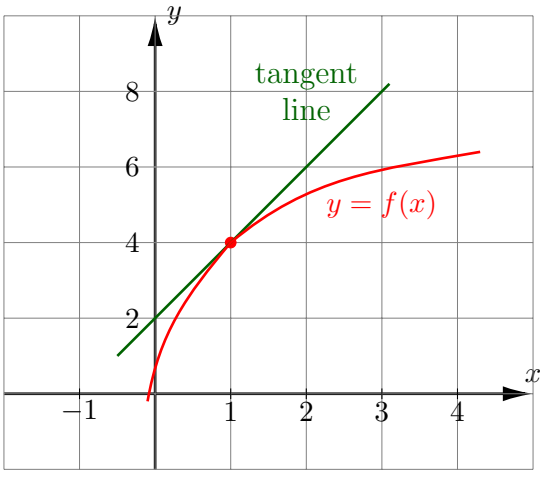

Figure 2

9.(5 pts.) If $g(x)$ is the function whose graph is given in Figure 3, then compute the derivative of the function $e^{g(x)}$ at $x = 6$.

(a) $2e^6$

(b) e^2

(c) $2e^5$

(d) e^4

(e) e^6

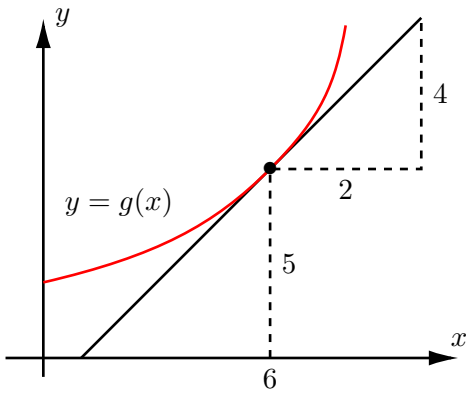

Figure 3

10.(5 pts.) If $f(x) = x^4 - 2x^3 + 2x^2 + 8000$ then $f''(1)$ is:

(a) -18

(b) 2

(c) 6

(d) 4

(e) 0

11.(5 pts.) The graph of the function $f(x)$ is given in Figure 4 below. The points in the open interval $(0, 15)$ where $f(x)$ is **not** differentiable are:

(a) $2, 5, 9, 13$

(b) $2, 8, 13$

(c) $2, 5, 8, 9, 13$

(d) $2, 5$

(e) $2, 5, 13$

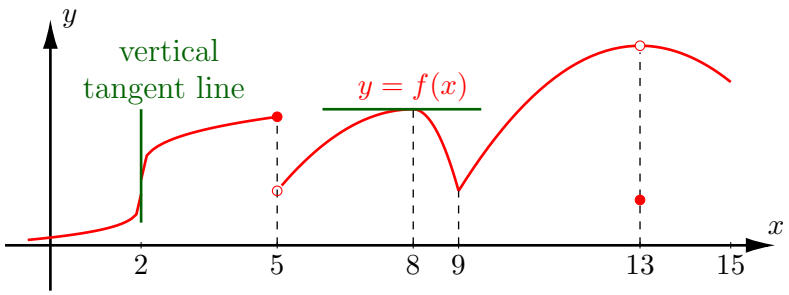

Figure 4

12.(5 pts.) Given that $f(1) = 2$ and $f'(1) = 3$, find the instantaneous rate of change of the quantity $\dfrac{f(t)}{t^3 - t + 1}$ at $t = 1$.

(a) 1 (b) $-\dfrac{1}{2}$ (c) 0 (d) 2 (e) -1

13.(5 pts.) The function $y = f(x)$ satisfies the following properties:
(i) The graph of $f(x)$ has a vertical asymptote at $x = -1$
(ii) The graph of $f(x)$ has a horizontal asymptote at $y = 1.5$
(iii) $f'(x) < 0$ on $(-\infty, -1)$ and $(1, \infty)$, and $f'(x) > 0$ on $(-1, 1)$
(iv) $f''(x) < 0$ on $(-\infty, -1)$ and $(-1, 2)$, and $f''(x) > 0$ on $(2, \infty)$.
Which of the graphs labeled (I) through (IV) could be the graph of $f(x)$?

(a) I (b) II (c) III (d) IV (e) I and III

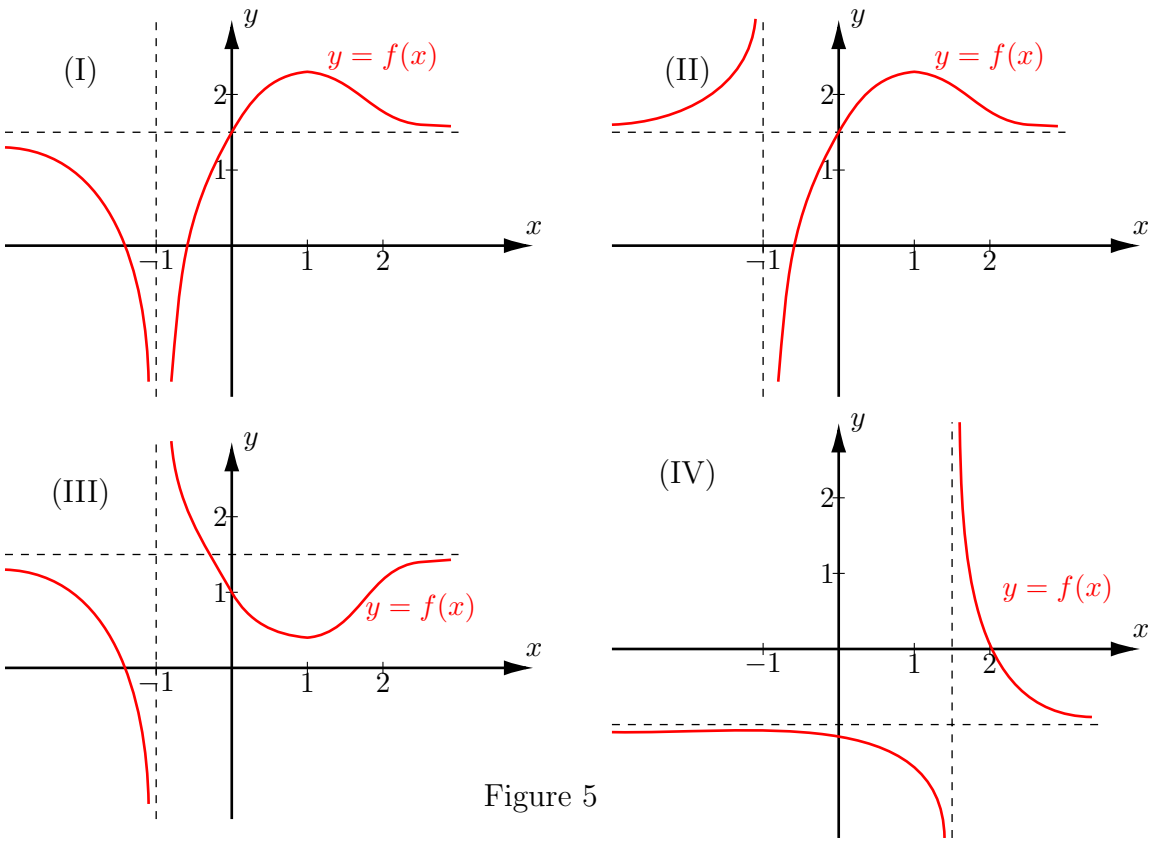

Figure 5

14.(5 pts.) Consider the curve $xe^y + ye^x = 1$. Find $\dfrac{dy}{dx}$.

(a) $\dfrac{dy}{dx} = \dfrac{1}{xe^y + e^y}$ (b) $\dfrac{dy}{dx} = xe^y + ye^x$ (c) $\dfrac{dy}{dx} = \dfrac{-e^y - ye^x}{xe^y + e^x}$

(d) $\dfrac{dy}{dx} = \dfrac{-xe^y - e^x y}{e^y + ye^x}$ (e) $\dfrac{dy}{dx} = \dfrac{-xe^y - e^y - ye^x}{e^x}$

15.(5 pts.) An oil slick in the shape of a circle is spreading on the surface of the ocean. When the radius of the slick is 50 yards, measurements indicate the radius is increasing at a rate of 8 yards per minute. How fast is the area of the slick increasing at this moment? (Recall that the area A of a circle with radius r is given by $A = \pi r^2$.)

(a) 700π

(b) 900π

(c) 500π

(d) 800π

(e) 600π

16.(5 pts.) Eleanor has 20 feet of fencing and wants to build a rectangular enclosure. What is the largest area that she can enclose?

(a) 24 ft^2

(b) 25 ft^2

(c) 8 ft^2

(d) 20 ft^2

(e) 100 ft^2

17.(5 pts.) Find the inflection point(s) of $f(x) = x^5 + 5x^4 + 8$, if any. (Hint: Make sure that concavity changes at each of your candidates for inflection points.)

(a) $x = -3, 0$

(b) $x = -3$

(c) $x = 3$

(d) $x = 4, 0$

(e) $x = 4$

18.(5 pts.) Suppose $C(x) = 80 + 10x + \dfrac{90}{x+1}$ is the cost function for a certain production process, where x is the production level in hundreds of units ($x \geq 0$) and $C(x)$ is the cost in thousands of dollars. What is the minimum possible cost?

(a) \$110,000

(b) \$150,000

(c) \$130,000

(d) \$140,000

(e) \$120,000

19.(5 pts.) A Frisbee is thrown to the point F, 8 yards inside a lake, as shown in Figure 6. At that time a dog, located at the point D on the beach 80 yards from A, starts running toward it along the beach to a point B and then swimming to the point F of the floating Frisbee. If the dog can run along the beach with speed 100 yards per minute and swim inside the lake with speed 10 yards per minute, then find the function that one should minimize to determine the location of the point B, which leads the dog to the Frisbee the fastest. (**Do not minimize it!**)

(a) $f(x) = \dfrac{80-x}{10} + \dfrac{\sqrt{x^2+64}}{100}$

(b) $f(x) = \dfrac{80-x}{100} + \dfrac{\sqrt{x^2+64}}{10}$

(c) None of these

(d) $f(x) = \dfrac{80-x}{100} - \dfrac{\sqrt{x^2+64}}{10}$

(e) $f(x) = 100(80-x) + 10\sqrt{x^2+64}$

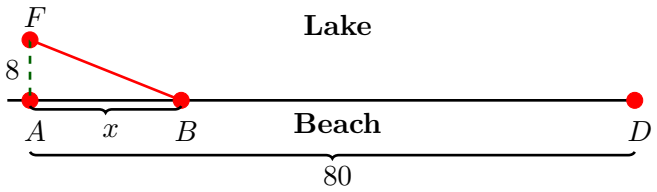

Figure 6

20.(5 pts.) Which of the following statements is **FALSE** for the function $f(x)$ whose graph is displayed in Figure 7?

(a) The area between the graph of $f(x)$ and the x-axis for $-2 \le x \le 4$ is equal to 8.

(b) $\int_0^2 f(x)dx = 2$

(c) $\int_2^4 f(x)dx = 4$

(d) $\int_{-2}^0 f(x)dx = -2$

(e) $\int_{-2}^4 f(x)dx = 8$

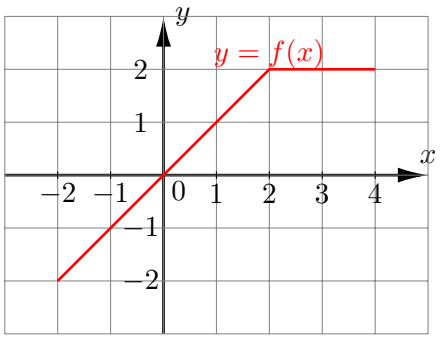

Figure 7

21.(5 pts.) Find $\int (\ln x) \cdot x^8 dx$.

(a) $\frac{1}{9}x^9 \ln x + c$

(b) $\frac{1}{8}x^8 \ln x - \frac{1}{64}x^8 + c$

(c) $\frac{1}{81}x^9 + c$

(d) $\frac{1}{9}x^9 \ln x + x^9 + c$

(e) $\frac{1}{9}x^9 \ln x - \frac{1}{81}x^9 + c$

22.(5 pts.) Let $f(x)$ be the function whose graph is given in Figure 8. Estimate the integral $\int_0^2 f(x)dx$ by using the Riemann sum corresponding to $n = 4$ and midpoints.

(a) -1.5

(b) 1.5

(c) 2

(d) 0

(e) 9

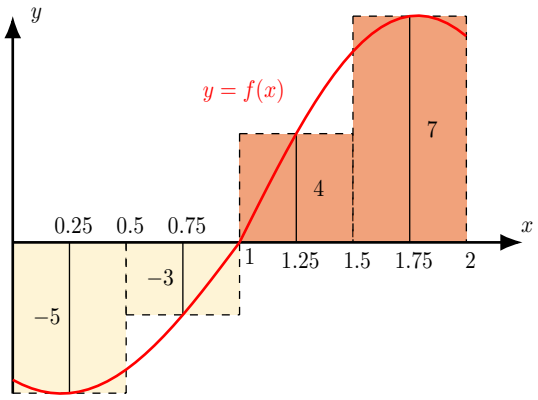

Figure 8

23.(5 pts.) Find the following definite integral $\int_0^1 (x^3+1)e^{x^4+4x}dx$.

(a) $\frac{1}{4}(e^5-1)$

(b) e^5-1

(c) $\frac{4}{3}e-1$

(d) $\frac{3}{4}e^4$

(e) $4(e^5-1)$

24.(5 pts.) The graph of each of the functions $f(x)$ and $g(x)$ is given in Figure 9. The area enclosed between them from $x=1$ to $x=7$ is given by:

(a) $\int_1^5[f(x)-g(x)]dx+\int_5^7[f(x)-g(x)]dx$

(b) $\int_1^7[f(x)-g(x)]dx$

(c) $\int_1^5[f(x)-g(x)]dx+\int_5^7[g(x)-f(x)]dx$

(d) None of these

(e) $\int_1^7[g(x)-f(x)]dx$

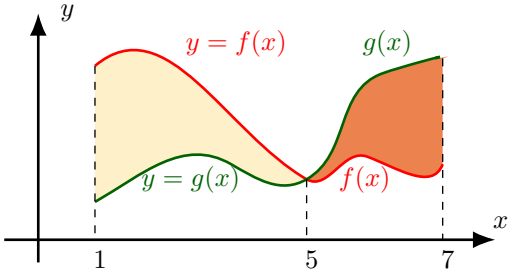

Figure 9

25.(5 pts.) The marginal profit of a company for producing and selling x units of an item is given by $MP(x) = -0.4x + 20$. Determine the change in profit if the company changes its production level from 40 to 50 units.

(a) 50

(b) 60

(c) 40

(d) 30

(e) 20

26.(5 pts.) The marginal revenue $MR = R'(x)$ from the sales of a certain item is shown in Figure 10. Here, x is in thousands of units and R is in thousands of dollars. Using the midpoint rule with $n = 5$, approximate the total revenue from the sales of the first thousand units.

(a) 50

(b) 30

(c) 150

(d) 40

(e) 75

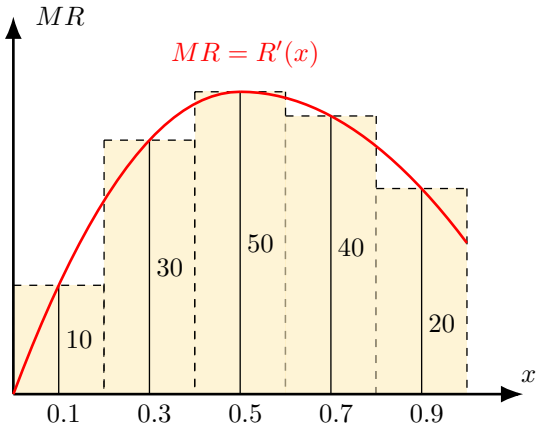

Figure 10

27.(5 pts.) Suppose you deposit \$8,000 in an account paying 4% annual interest rate, compounded continuously, and do not make any further deposits or withdrawals. Find the **average** amount of money in the account during the first 5 years.

(a) None of these

(b) $\int_0^5 8000e^{0.04t}dt$

(c) $\frac{1}{5}\int_0^5 8000e^{-0.04t}dt$

(d) $\int_0^5 8000e^{-0.04t}dt$

(e) $\frac{1}{5}\int_0^5 8000e^{0.04t}dt$

28.(5 pts.) Let $f(x)$ be the function whose graph is given in Figure 11. Then for the function $F(t) = \int_0^t f(x)dx$ choose the statement below which is **NOT** true.

(a) $F'(8) = 0$

(b) $F(6) = 16$

(c) $F(8) = 40$

(d) $F'(3) = 5$

(e) $F(3) = 9$

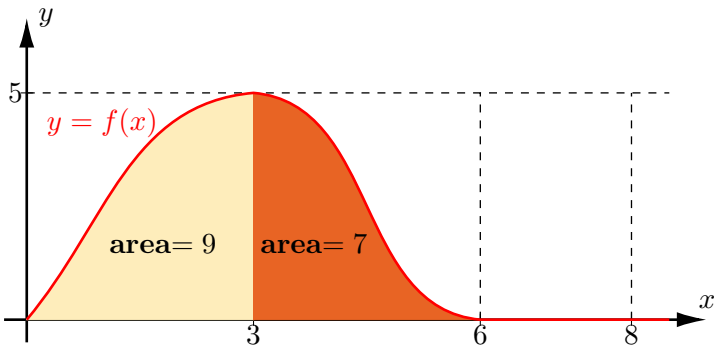

Figure 11

29.(5 pts.) Assume that for a certain commodity the demand curve is

$$D(q) = \frac{16}{q+1}$$

and the supply curve is $S(q) = q + 1$. Given that the equilibrium quantity is $q_e = 3$ and the equilibrium price is $p_e = 4$, determine the consumer surplus (CS).

(a) $16 \ln 2 - 12$

(b) $16 \ln 4$

(c) $16 \ln 4 + 12$

(d) $8 \ln 4 - 12$

(e) $16 \ln 4 - 12$

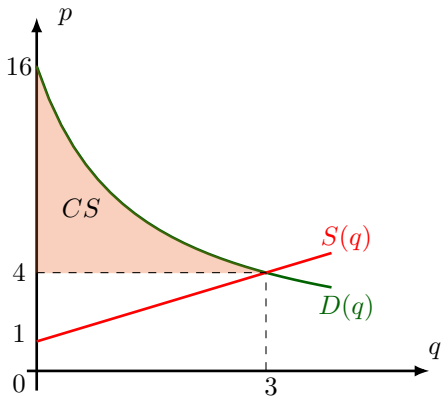

Figure 12

30.(5 pts.) Imagine that you just got that great job, which in addition to a salary it also offers you the following IRA option. For the next 20 years it deposits money continuously into an account at a rate of $2{,}500e^{0.2t}$ dollars per year. Find the present value of this IRA account assuming that it earns an annual interest of 10% compounded continuously.

(a) 25,000

(b) $25{,}000(e^2 + 1)$

(c) 80,000

(d) $25{,}000(e^2 - 1)$

(e) 50,000

Name: _____

Practice A, Math 10250, Final Exam Instructor: ANSWERS

- Be sure that you have all 16 pages of the test.
- Calculators are allowed for this examination.
- The exam lasts for .
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- **Sign the pledge.** "As a member of the Notre Dame Community, I will not participate in or tolerate academic dishonesty":

_____

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(•)	17.	(a)	(•)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(•)	(d)	(e)	19.	(a)	(•)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(•)	(e)	20.	(a)	(b)	(c)	(d)	(•)
5.	(•)	(b)	(c)	(d)	(e)	21.	(a)	(b)	(c)	(d)	(•)
6.	(a)	(•)	(c)	(d)	(e)	22.	(a)	(•)	(c)	(d)	(e)
7.	(a)	(b)	(•)	(d)	(e)	23.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)	24.	(a)	(b)	(•)	(d)	(e)
9.	(a)	(b)	(•)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(•)
10.	(a)	(b)	(c)	(•)	(e)	26.	(a)	(•)	(c)	(d)	(e)
11.	(•)	(b)	(c)	(d)	(e)	27.	(a)	(b)	(c)	(d)	(•)
12.	(a)	(b)	(c)	(d)	(•)	28.	(a)	(b)	(•)	(d)	(e)
13.	(•)	(b)	(c)	(d)	(e)	29.	(a)	(b)	(c)	(d)	(•)
14.	(a)	(b)	(•)	(d)	(e)	30.	(a)	(b)	(c)	(•)	(e)
15.	(a)	(b)	(c)	(•)	(e)						
16.	(a)	(•)	(c)	(d)	(e)						