

Department of Mathematics
University of Notre Dame
Math 10250 – Elem. of Calc. I

Name: _____

Instructor: _____

Practice A – Exam 2

This exam is in 2 parts on 10 pages and contains 14 problems worth a total of 100 points. You have to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached. Good luck!

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Place an \times through your answer to each problem.

- | | | | | | |
|-----|-----|-----|-----|-----|-----|
| 1. | (a) | (b) | (c) | (d) | (e) |
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14. _____

Tot. _____

Multiple Choice

1. (5 pts.) Recall that the magnitude of an earthquake is measured using the Richter scale. An earthquake having seismic waves of amplitude x has Richter value $\log_{10} \frac{x}{A}$, where A is a reference amplitude.

A powerful earthquake of magnitude 7.5 rocked the Indonesian island of Sulawesi on Sep 28, 2018. In 2011, an undersea earthquake of magnitude 9.1 triggered a major tsunami in Japan causing a nuclear accident. The Japan earthquake was **P** times stronger than the Indonesia earthquake. Then **P** is

- (a) $10^{1.6}$
- (b) $\ln \frac{9.1}{7.5}$
- (c) $\frac{\log_{10} 9.1}{\log_{10} 7.5}$
- (d) $e^{1.6}$
- (e) $\log_{10} \frac{9.1}{7.5}$

2. (5 pts.) Evaluate exactly $\lim_{h \rightarrow 0} \frac{(e+h) \ln(e+h) - e}{h}$. (**Hint:** Think Derivative.)

- (a) $\frac{1}{e}$
- (b) e
- (c) 2
- (d) 1
- (e) -2

3. (5 pts.) The tables below represent the Revenue and Cost data for a certain car company.

Revenue	x	100	120	140	160	180	200	220
	$R(x)$	5.5	6.8	8.1	9.4	10.8	12.3	13.8

Cost	x	100	120	140	160	180	200	220
	$C(x)$	4.5	6.0	7	7.8	8.5	9.1	9.6

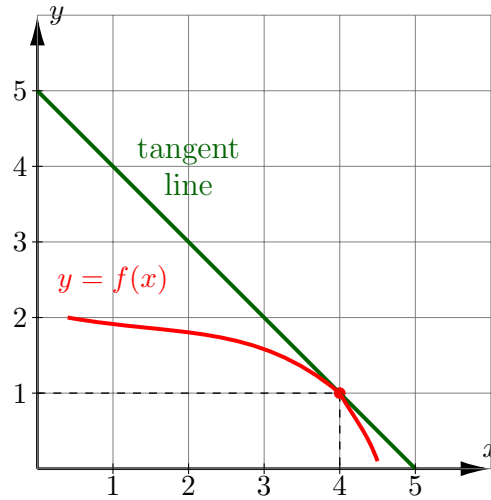
where x denotes the number cars sold per month. Here, $R(x)$ and $C(x)$ are in millions of dollars. Then the marginal profit at $x = 220$, using the backward difference formula, is approximately equal to

- (a) 0.01
- (b) 0.02
- (c) 0.50
- (d) 0.05
- (e) 0.04

4. (5 pts.) $\frac{d^3}{dx^3}(xe^{x+1})$ is

- (a) $2e^{x+1} + 3xe^{x+1}$
- (b) $2e^{x+1} + 3e^{x+1}$
- (c) $3e^{x+1} + 2xe^{x+1}$
- (d) $3e^{x+1} + xe^{x+1}$
- (e) $e^{x+1} + 3xe^{x+1}$

5. (5 pts.) The following represents the graph of a function $f(x)$. Let $G(x) = f(x)e^{f(x)}$. Then $G'(4)$ is



- (a) $4e$
 (b) $-8e$
 (c) $-2e$
 (d) $2e$
 (e) $8e$
6. (5 pts.) Suppose an unknown amount is deposited in a bank account paying 9% annual interest, **compounded continuously**. How many years will it take for the balance to triple?

- (a) e^3
 (b) $\frac{0.09}{\ln 30}$
 (c) $\frac{0.09}{\ln 3}$
 (d) $3e^{0.9}$
 (e) $\frac{\ln 3}{0.09}$

7. (5 pts.) An intravenous drug was given to a patient on **Monday** at 12am. Suppose on Tuesday at 12am, the amount of drug present in the body was 400ml, and the drug was leaving the body at the rate of 20ml/day. Then the amount of drug present inside the body at 6pm on **Monday** was
(**Hint:** Linear approximation.)

- (a) 410 ml.
- (b) 405 ml.
- (c) 404 ml.
- (d) 390 ml.
- (e) 396 ml.

8. (5 pts.) Thorium-234 is a radioactive substance with a half-life of 24.5 days. How many days will it take for an initial amount to decrease to one-fourth of its mass?

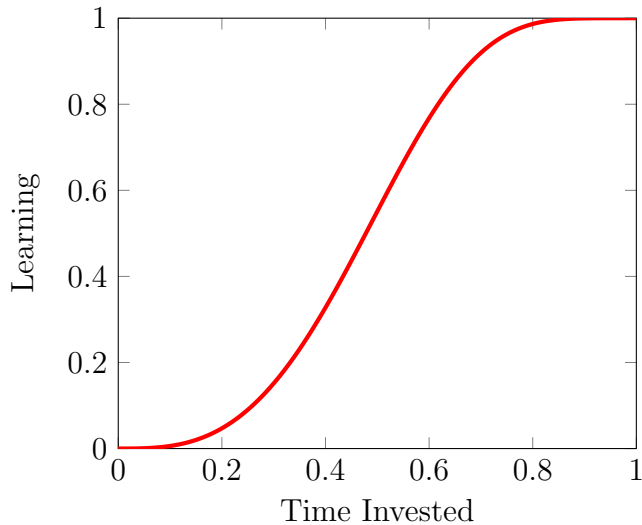
- (a) $\left(\frac{\ln 2}{\ln 4}\right) 24.5$
- (b) $\left(\frac{\ln 4}{\ln 5}\right) 24.5$
- (c) 80
- (d) $\left(\frac{\ln 3}{\ln 4}\right) 24.5$
- (e) 49

9. (5 pts.) Suppose that $\ln A = \frac{1}{10}$. Compute exactly $A^5 \ln \left(\frac{1}{A} \right)$.

- (a) $-\frac{1}{10}e^2$
- (b) $\frac{1}{10}e^2$
- (c) $\frac{2}{10}e^{\frac{1}{2}}$
- (d) $-\frac{1}{10}e^{\frac{1}{2}}$
- (e) $\frac{1}{10}e^{\frac{1}{2}}$

10. (5 pts.) A **learning curve** is a graphical representation of how an increase in learning comes from time invested. The graph below is a special example of a learning curve called the *Sigmoidal Curve*. For the sigmoidal learning curve, which of the following statements is **False**?

S-Curve (Sigmoid)



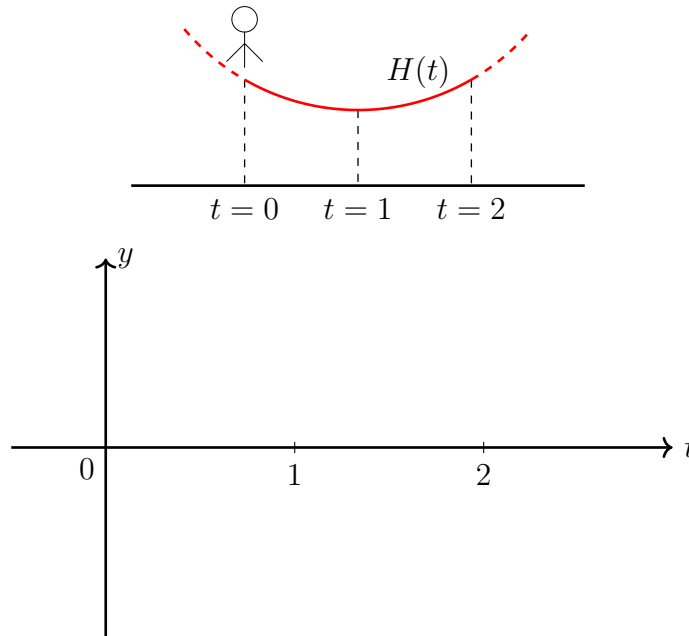
- (a) The more someone does something, the better they get at it.
- (b) The rate of change of learning is not constant.
- (c) Learning is a **linear** function of time invested.
- (d) The rate of change of learning reaches a maximum at $t = 0.5$.
- (e) Learning is an **increasing** function of time invested.

Partial Credit

You must **show your work** on the partial credit problems to receive credit!

11. (12 pts.) [**Show your work**]

(A) (6 pts.) A child plays in a circular swing (as shown in the picture) and suppose at time $t = 0$, she starts to slide down. Let $H(t)$ denote the height of her position from the ground at time t . Plot the graph of $H'(t)$ with respect to time.



(B) (Independent of A.)(6pts.) A ball is thrown into the air and its height in feet (measured from the ground) after t seconds is given by $s = -16t^2 + 64t + 80$ until it hits the ground.

(a) What is the velocity at the end of 1 sec? In what direction (up or down) is it going at the end of 1 sec?

Answer: _____

(b) What is the ball's acceleration at the end of 1 sec? What is the ball's acceleration at the end of 3 secs?

Answer: _____

12. (12 pts.) [**Show your work**]

(A) (6pts.) Using the **definition** of the derivative, find the derivative of $f(x) = \frac{1}{5x - 2}$.

Answer: _____

(B) (6pts.) Estimate $\sqrt{95}$ using linear approximation.

Answer: _____

13. (12 pts.) [**Show your work**]

(A) (6pts) A jar of grape jelly has fungus growing inside it, since it has been left opened. Suppose, today the population of the fungus-causing bacteria is 8000, and 10 days later it is predicted to reach 16000. Assume that the rate of change of the population is directly proportional to the population itself.

(a) Find a formula for $P(t)$, the population of the bacteria as a function of time.

Answer: _____

(b) Assume 500 bacteria had entered the jar the moment it was opened for the first time. How many days ago, was the jar opened?

Answer: _____

(B) (Independent of A.)(6pts). Compute the derivative of the function $f(x) = \frac{x}{e^{2x} + 5} + \pi^x$.

Answer: _____

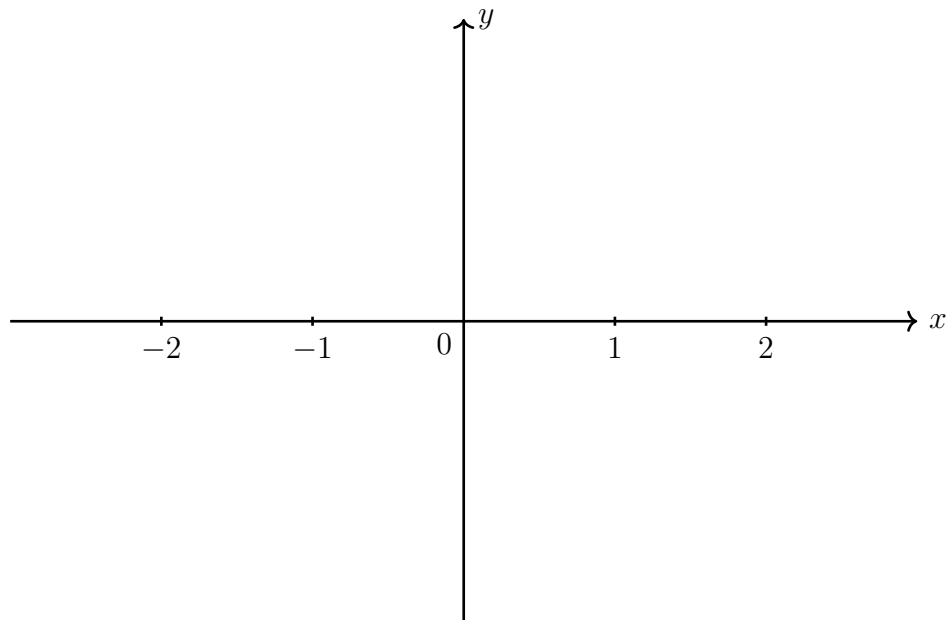
14. (14 pts.) [Show your work]

(A) (8pts). At 6pm, a roast turkey is taken out from an oven when the temperature of the turkey has reached 200F and is placed on a table. Suppose that the room temperature is 70F, and the temperature of the turkey is 170F at 7pm. What will be the temperature of the turkey at 8pm?

Answer: _____

(B) (Independent of A.)(6pts.) Draw the graph of a function f on $[-2, 2]$ with the following properties:

- a) f is continuous everywhere on $[-2, 2]$, except at $x = 0$.
- b) f is not differentiable at $x = -1$, $x = 0$ and $x = 1$.



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