Practice A – Math 10250 Exam 2 Solutions

1.) Let x_J and x_I denote the amplitudes of the Japan and Indonesia earthquakes respectively. Then we have $9.1 = \log_{10}\left(\frac{x_J}{A}\right) \implies \frac{x_J}{A} = 10^{9.1}$ and $7.5 = \log_{10}\left(\frac{x_I}{A}\right) \implies \frac{x_I}{A} = 10^{7.5}$. Therefore $\frac{x_J}{x_I} = 10^{9.1-7.5} \implies x_J = 10^{1.6}x_I$. Thus $P = 10^{1.6}$.

2.) Notice that $\lim_{h\to 0}\dfrac{(e+h)\ln(e+h)-e}{h} = f'(e)$, where $f(x) = x\ln x$. But $(x\ln x)' = \ln x + x\cdot\frac{1}{x} = \ln x + 1$. Therefore $\lim_{h\to 0}\dfrac{(e+h)\ln(e+h)-e}{h} = \ln e + 1 = 2$.

2.) The backward difference formula applied to Revenue and Cost functions, gives $MR(220) = \frac{13.8-12.3}{20} = \frac{1.5}{20}$ and $MC(220) = \frac{9.6-9.1}{20} = \frac{.5}{20}$. Thus the marginal profit is $MP(220) = MR(220) - MC(220) = \frac{1}{20} = 0.05$.

6.) If $f(x) = xe^{x+1}$, then $f'(x) = e^{x+1} + xe^{x+1}$ and $f''(x) = e^{x+1} + \left(xe^{x+1}\right)' = e^{x+1} + e^{x+1} + xe^{x+1}$ and $f'''(x) = e^{x+1} + e^{x+1} + e^{x+1} + xe^{x+1} = 3e^{x+1} + xe^{x+1}$.

5.) From the graph, it is clear that $f(4) = 1$ and $f'(4) = -1$. Now $G'(x) = \left(f(x)e^{f(x)}\right)' = f'(x)e^{f(x)} + f(x)e^{f(x)}f'(x)$. Thus $G'(4) = f'(4)e^{f(4)} + f(4)e^{f(4)}f'(4) = -1e^1 + 1e^1(-1) = -2e$.

6.) Let the unknown initial amount deposited be A. Since we are compounding continuously, the account balance follows the equation $A(t) = Ae^{.09t}$. Suppose the balance triples in time t. Then we have $3A = Ae^{.09t}$. Taking ln on both sides and solving for t, we have $t = \frac{\ln 3}{.09}$ years.

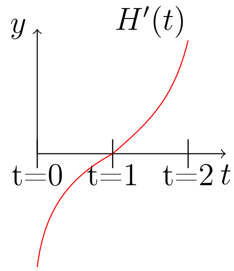
7.) Let $D(t)$ denote the amount of drug present after t days. Then we have $D(1) = 400$ and $D'(1) = -20$. Note that in terms of days, 6pm on Monday corresponds to 18hrs which is equivalent to .75 days. Linear approximation at $t = 1$ is given by the equation $L(t) = D(1) + D'(1)(t-1)$ or $L(t) = 400 + -20(t-1)$. Plugging in $t = 0.75$, we have $L(0.75) = 400 + -20(0.75-1) = 400 + 5 = 405$.

8.) The rate of decay k satisfies the formula $k = \frac{\ln 2}{\text{Half Life}}$. We have $y(t) = y_0e^{-kt}$, where y_0 denotes the initial amount present. Suppose the amount reduces to one-fourth in time t. Then $\frac{1}{4}y_0 = y_0e^{-\frac{\ln 2}{24.5}t}$. Taking ln on both sides and simplifying we obtain $t = 24.5\frac{\ln 4}{\ln 2} = 24.5\frac{2\ln 2}{\ln 2} = 24.5(2) = 49$ years.

9.) Using the formulas $A^B = e^{B\ln A}$ and $\ln\left(\frac{1}{A}\right) = -\ln A$, we see that $A^5\ln\left(\frac{1}{A}\right) = e^{5\ln A}(-\ln A) = e^{5\frac{1}{10}}\left(-\frac{1}{10}\right) = -\frac{1}{10}e^{\frac{1}{2}}$.

10.) Learning is clearly not a linear function of time invested.

11.) (A) By a quick visual inspection, we see that $H'(0)$ is negative, $H'(1)$ =0, and $H'(2)$ is positive. We also notice that $H'(t)$ is an increasing function in t. Hence one possible graph of $H'(t)$ is as follows



(B) (a) Velocity $v(t)$ at any time is given by $s'(t) = -32t + 64$. Hence $v(1) = 32$, which has positive sign, thus the ball is going up. (b) Acceleration at any time is given by $a(t) = v'(t) = -32$. Therefore, at all times the acceleration is constant.

12.) (A) From the definition of derivative, $f'(x) = \lim_{h\to 0} \frac{\frac{1}{5(x+h)-2} - \frac{1}{5x-2}}{h} = \lim_{h\to 0} \frac{\frac{5x-2-5(x+h)+2}{(5(x+h)-2)(5x-2)}}{h}$ or f'(x)=$\lim_{h\to 0} \frac{\frac{-5h}{(5(x+h)-2)(5x-2)}}{h}$. Further simplification leads to $f'(x) = \lim_{h\to 0} \frac{-5}{(5(x+h)-2)(5x-2)}$ which is equal to $\frac{-5}{(5x-2)(5x-2)}$ or $-5\,(5x-2)^2$.

(B) We use the linear approximation near $x = 100$ in the funtion $f(x) = \sqrt{x}$. Since $f'(x) = \frac{1}{2\sqrt{x}}$, we have $f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$. Thus linear approximation near 100 is given by $L(x) = f(100) + f'(100)(x - 100)$ or $L(x) = 10 + \frac{1}{20}(x - 100)$. Plugging in $x = 95$, we obtain $\sqrt{95} \approx L(95) = 10 - \frac{5}{20}$.

13.) (A) (a) The bacteria population satisfies $P(t) = 8000e^{kt}$. We use $P(10) = 16000$, to obtain k from the equation: $16000 = 8000e^{10k}$. Taking ln on both sides, we have $10k = \ln 2 \implies k = \frac{1}{10}\ln 2$. So, $P(t) = 8000e^{\left(\frac{1}{10}\ln 2\right)t}$. (b) We solve for t in $500 = 8000e^{\left(\frac{1}{10}\ln 2\right)t}$, and obtain $t = -\left(\frac{\ln 16}{\ln 2}\right)10 = -40$. Thus the jar was opened 40 days ago.

(B) Using the quotient rule, we have $\left(\frac{x}{e^{2x}+5}\right)' = \frac{1\left(e^{2x}+5\right)-(x)\left(2e^{2x}\right)}{\left(e^{2x}+5\right)^2}$. Recall $(b^x)' = (\ln b)\,b^x$, thus $(\pi^x)' = (\ln \pi)\,\pi^x$. Therefore $\left(\frac{x}{e^{2x}+5} + \pi^x\right)' = \frac{1\left(e^{2x}+5\right)-(x)\left(2e^{2x}\right)}{\left(e^{2x}+5\right)^2} + (\ln \pi)\,\pi^x$.

14.) (A) Applying Newton's Law of cooling, the temperature function of the turkey is given by $H(t) = 70 + Ae^{kt}$. At $t = 0$, we have $H(0) = 200 = 70 + Ae^0 = 70 + A \implies A = 200 - 70 = 130$. We also have $H(1) = 170 = 70 + 130e$, therefore $130e^k = 100$. Taking ln on both sides we obtain $k = \ln\frac{10}{13}$. So the temperature function is given by $H(t) = 70 + 130e^{\left(\ln\frac{10}{13}\right)t}$. Plugging in $t = 2$, we see that the temperature at 8pm is $70 + 130e^{2\left(\ln\frac{10}{13}\right)}$.

(B) The graph below satisfies (a) and (b).