Practice A – Math 10250 Exam 1 Solutions

1.) We need to find the equation of the line passing through the points $(t_1 = 0,\ E_1 = 20,000)$ and $(t_2 = 5,\ E_2 = 140,000)$. The slope of this line is $\frac{\Delta E}{\Delta t} = \frac{140,000-20,000}{5-0} = \frac{120,000}{5} = 24,000$. Therefore using the point-slope formula we get $E - 20,000 = 24,000(t-0)$, or $E = 24,000t + 20,000$.

2.) The demand function is decreasing and the supply function is increasing. To find the equilibrium point we set demand equal to suply, that is we solve the equation $-\dfrac{1}{2}p + 8 = \dfrac{2}{3}p - \dfrac{4}{3}$, which gives $8 + \dfrac{4}{3} = \left(\dfrac{2}{3} + \dfrac{1}{2}\right)p$ or $p = 8$. Setting $p = 8$ into demand (or supply) we get $q = -\dfrac{1}{2}\cdot 8 + 8 = 4$. Thus the the equilibrium $(p_e, q_e) = (8, 4)$.

3.) Using the identity $A^2 - B^2 = (A-B)(A+B)$ we have

$$P(x) = -10(x-12)^2 + 250 = -10[(x-12)^2 - 5^2] = -10[(x-12-5)(x-12+5)] = -10(x-17)(x-7),$$

which shows that $P(x) > 0$ if $7 < x < 17$.

4.) Using the identity $(A+B)^2 = A^2 + 2AB + B^2$ we have

$$\lim_{h\to 0}\frac{5(3+h)^2 - 45}{h} = \lim_{h\to 0}\frac{5(9+6h+h^2) - 45}{h} = \lim_{h\to 0}\frac{30h + 5h^2}{h} = \lim_{h\to 0}(30 + 5h) = 30 + 0 = 30.$$

5.) We have

$$\lim_{x\to\infty} R(x) = \lim_{x\to\infty}\frac{400x + 1200}{2x+5} = \lim_{x\to\infty}\frac{400 + 1200/x}{2 + 5/x} = \frac{400 + \lim_{x\to\infty}[1200/x]}{2 + \lim_{x\to\infty}[5/x]} = \frac{400+0}{2+0} = 200.$$

Thus, if the company keeps spending more and more money in advertising then the revenue's limiting value is \$200 million.

6.) Writing $f(x) = \dfrac{x-5}{(x-5)(x-3)} \overset{x\neq 5}{=} \dfrac{1}{x-3}$, we see that $x = 3$ is a vertical asymptote since $\lim\limits_{x\to 3^\pm}\dfrac{1}{x-3} = \pm\infty$. Also, we have that $y = 0$ is a horizontal asymptote, since $\lim\limits_{x\to\pm\infty}\dfrac{1}{x-3} = 0$. The natural domain of the function $f(x)$ consists of all numbers except $x = 3$ and $x = 5$, which are the zeros of the denominator.

7.) Since temperature is a continuous function of time and the value 63 is between $H(1) = 64$ and $H(3) = 55$, and also between $H(7) = 61$ and $H(9) = 70$, by the intermediate value theorem the temperature assumes the value 63 in the time intervals $[1, 3]$ and $[7, 9]$ for certain.

8.) The function $f(x)$ is continuous everywhere except possibly at $x = 1$, which is the zero of the denominator. Since $\lim\limits_{x\to 1}\dfrac{x^2 + 6x - 7}{x-1} = \lim\limits_{x\to 1}\dfrac{(x-1)(x+7)}{x-1} = \lim\limits_{x\to 1}(x+7) = 8$, we see that $f(x)$ is continuous at $x = 1$ too if we define $f(1) = 8$, or choose $c = 8$.

9.) The function $f(x)$ is **not** continuous at $x = 4$ since its value there is 50, which is different from its limit as $x \to 4$. Note that this limit is 20.

10.) Looking at the graph of $f(x)$ we see that $\lim_{x\to 4} f(x) = 20$. Therefore, applying the limit laws we have

$$\lim_{x\to 4}\frac{\sqrt{xf(x) + 20}}{x^2 - 3x + 1} = \frac{\lim_{x\to 4}\sqrt{xf(x)+20}}{\lim_{x\to 4}[x^2 - 3x + 1]} = \frac{\sqrt{\lim_{x\to 4} x\cdot\lim_{x\to 4} f(x) + 20}}{\lim_{x\to 4} x^2 - 3\lim_{x\to 4} x + 1} = \frac{\sqrt{4\cdot 20 + 20}}{4^2 - 3\cdot 4 + 1} = \frac{10}{5} = 2.$$

11.) (i) First we solve the equation $y = \dfrac{x+2}{x+1}$ for x. For this we multiply the equation by $x+1$ and get $xy + y = x + 2$, or $xy - x = 2 - y$, or $(y-1)x = 2 - y$, or $x = \dfrac{2-y}{y-1}$. Next, we interchange x and y and obtain $y = \dfrac{2-x}{x-1}$. Thus the inverse of $f(x)$ is given by the function $g(x) = \dfrac{2-x}{x-1}$. Observe the natural domain of $f(x)$ consists of all numbers $x \neq -1$.

(ii) In this case the slope is equal to $\Delta q/\Delta p = (-1000)/5 = -200$. Moreover, when $p = 20$ then $q = 5000$. Therefore, using the point-slope formula we get the equation: $q - 5000 = -200(p-20)$, or $q = -200p + 9000$.

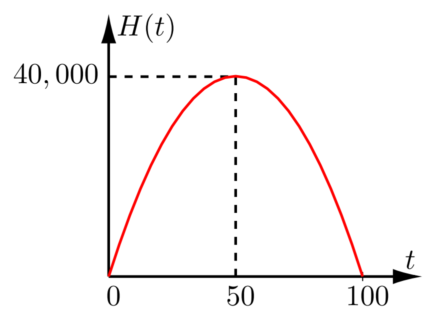
12.) (Ai) The revenue function is: $R = x \cdot q = x(-0.5x + 95) = -0.5x^2 + 95x$.

(Aii) The profit function is: $P = R - C = -0.5x^2 + 95x - 10q - 5000$. Taking the expression of q from demand and substituting it to the profit function we get: $P = -0.5x^2 + 95x - 10(-0.5x + 95) - 5000 = -0.5x^2 + 100x - 5950$.

(B) Using the formula $FV = PV\left(1 + \dfrac{r}{n}\right)^{nt}$, with $PV = 10{,}000$, $r = 0.05$, $n = 365$ and $t = 5$ we find the future value $FV = 10{,}000\left(1 + \dfrac{0.05}{365}\right)^{365 \cdot 10} \approx 16{,}487$ dollars.

13.) (i) We have

$$
\begin{aligned}
H(t) &= -16t^2 + 1600t \\
&= -16[t^2 - 100t] \\
&= -16[t^2 - 2 \cdot t \cdot 50] \\
&= -16[t^2 - 2 \cdot t \cdot 50 + 50^2 - 50^2] \\
&= -16[(t-50)^2 - 50^2] \\
&= -16(t-50)^2 + 16 \cdot 50^2 \\
&= -16(t-50)^2 + 40{,}000
\end{aligned}
$$



(Note that there are other ways for completing square.)

(ii) Since $H(t) = -16(t-50)^2 + 40{,}000$ we see that $H(t)$ takes its maximum value 40,000 at $t = 50$.

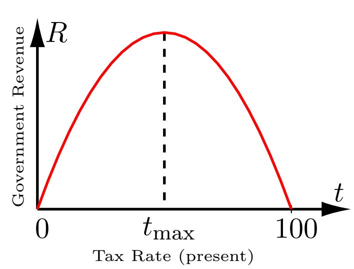
(iii) The rock hits the ground when $H(t) = 0$, or $16(t-50)^2 + 40{,}000 = 0$, or $(t-50)^2 = 40{,}000/16$, or $t - 50 = \pm 50$, or $t = 100$ seconds.

14.) (A) For $h \neq 0$ we have $\dfrac{\sqrt{25+h}-5}{h} = \dfrac{(\sqrt{25+h}-5)(\sqrt{25+h}+5)}{h(\sqrt{25+h}+5)} = \dfrac{\not h}{\not h(\sqrt{25+h}+5)} = \dfrac{1}{\sqrt{25+h}+5}$.

Therefore, the given limit is equal to $\lim_{h\to 0} \dfrac{1}{\sqrt{25+h}+5} = \dfrac{1}{\sqrt{25}+5} = \dfrac{1}{10} = 0.1$.

(B) We find the future value of federal debt in 10 years by applying the formula $FV = PVe^{rt}$ with $PV = 21.5$, $r = 0.04$ and $t = 10$. Thus, we have $FV = 21.5e^{0.04 \cdot 10} = 21.5e^{0.4} \approx 32.07$ trillion dollars.

(C) The **Laffer curve** looks like the parabola below:



Note: This curve was popularized by Arthur Laffer (b. 1940) though its underlying principle was widely known long before that since the time of Ibn Khaldun's Muqaddimah (1377). See wikipedia.org/wiki/Laffer_curve.