Name ______________________________ Date ______________________________

**Math 10250 Activity 29: The Indefinite Integral (Section 5.1)**

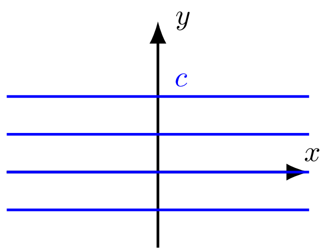
**GOAL:** If we are given the derivative $f'(x)$, we want to be able to find the function $f(x)$.

▶ **Antiderivatives** (Reversing differentiation)

**Definition:** $F'(x) = f(x)$ means that $F(x)$ is an **antiderivative** of $f(x)$.

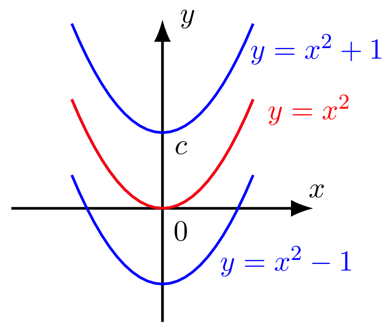
**Example 1** Find all the antiderivatives of the indicated function $f(x)$. That is, find all the functions $F(x)$ so that when we take their derivative, we get $f(x)$. In each case, sketch three of them on the same set of axes.

$f(x) = 0 \Longrightarrow F(x) = c$, *for any constant c*



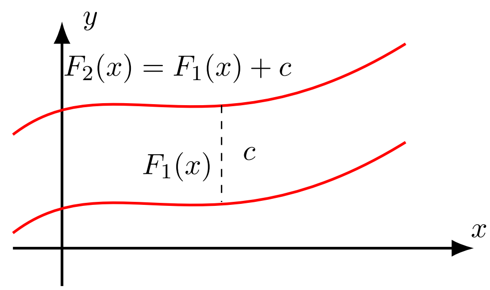

$f(x) = 2x$

$F(x) = \boxed{x^2 + c}$, $c = 0, \pm 1$, *any number*




From Example 1, we see that

> **Theorem** If $F_1(x)$ and $F_2(x)$ are antiderivatives of the same function throughout an interval, then they differ by a constant $c$ over that interval; that is, for $a < x < b$
> $F_1'(x) = F_2'(x) \Longrightarrow F_2(x) - F_1(x) = c$ or $F_2(x) = F_1(x) + c$
> for some number $c$.




**Q1:** How do we denote all antiderivatives of $f(x)$?

**A1:** If $F(x)$ is an antiderivative of $f(x)$; that is, $F'(x) = f(x)$. Then we may write

$$\int f(x)dx = F_1(x) + c, \text{ where } F_1 \text{ is an antiderivative}$$

We call $\int f(x)dx$ the **indefinite integral**.

**Example 2** Let $f(x) = (5x-1)^3$ and $F(x) = A(5x-1)^4$.

**a.** Find the value of the constant $A$ that makes $F(x)$ an antiderivative of $f(x)$.

$$F'(x) = A \cdot 4(5x-1)^3 \cdot 5 = 20A(5x-1)^3 \overset{must}{=\!=} (5x-1)^3 \Longrightarrow 20A = 1 \Longrightarrow \boxed{A = \frac{1}{20}}$$

**b.** Write your result in Part (a) in terms of the indefinite integral.

$\int (5x-1)^3 dx = \boxed{\frac{1}{20}(5x-1)^4 + c}$

**Example 3** Referring to Example 1, find the indefinite integral of $f(x) = 2x$.

$\int 2x dx = \boxed{x^2 + c}$

**Example 4** If $k \neq 0$, compute $\frac{d}{dx}\left(e^{kx}\right)$ then write down $\int e^{kx} dx$. (Use the fact: $(c \cdot f(x))' = c \cdot f'(x)$)

*Since* $\frac{d}{dx}(e^{kx}) = ke^{kx}$, *we have* $\int e^{kx} dx = \frac{e^{kx}}{k} + c$.

**Example 5** If $k \neq -1$, compute $\frac{d}{dx}\left(x^{k+1}\right)$ then write down $\int x^k dx$.

*Since* $\frac{d}{dx}(x^{k+1}) = (k+1)x^k$, *we have* $\int x^k dx = \frac{x^{k+1}}{k+1} + c$.

▶ **Basic indefinite integral formulas**

- For any constant $m$: $\boxed{\int m\, dx \stackrel{?}{=} mx + c.}$ For Example: $\int 100 dx \stackrel{?}{=} 100x + c$
- Power Rule when $k \neq -1$: $\boxed{\int x^k dx \stackrel{?}{=} \frac{1}{k+1} x^{k+1} + c.}$ For Example: $\int x^9 dx \stackrel{?}{=} \frac{1}{10} x^{10} + c$
- Power Rule when $k = -1$: $\boxed{\int \frac{1}{x}\, dx = \ln|x| + c.}$
- Exponential Rule: $\boxed{\int e^{kx} dx = \frac{1}{k} e^{kx} + c, k \neq 0}$ For Example: $\int e^{0.1x} dx \stackrel{?}{=} 10e^{0.1x} + c$
- Constant Multiple Rule: $\boxed{\int kf(x)dx = k \int f(x)dx, \text{ any } k}$ For Example: $\int \frac{8}{x} dx \stackrel{?}{=} 8\ln|x| + c$
- Sum Rule: $\boxed{\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx.}$

**Example 6** Find each of the following indefinite integrals. Check your answer by differentiation.

**a.** $\int \left(x^7 - 2x^{-4} + \frac{3}{x} + e^{2x}\right) dx = \frac{1}{7+1} x^{7+1} - 2\frac{1}{-4+1} x^{-4+1} + 3\ln|x| + \frac{1}{2} e^{2x} + c$

**b.** $\int \frac{3x - 10x^2 + \sqrt{x}}{x^3}\, dx = \int \left(3x^{-2} - 10\frac{1}{x} + x^{-\frac{5}{2}}\right) dx = 3\frac{1}{-2+1} x^{-2+1} - 10\ln|x| + \frac{1}{-5/2+1} x^{-\frac{5}{2}+1} + c$

**Example 7** Given that $\int f(x)dx = F(x) + c$ and $G'(x) = g(x)$. Find each of the following indefinite integrals in terms of $F(x)$, $G(x)$, and other known functions whenever possible. If not possible, state so.

**a.** $\int [2f(x) + 3x]\, dx$

$= 2\int f(x)dx + \frac{3}{2}x^2 + c$

$= 2F(x) + \frac{3}{2}x^2 + c$

**b.** $\int f(x) \cdot g(x)\, dx$

Not possible

**c.** $\int \frac{5 - 3x \cdot g(x)}{x}\, dx$

$= \int \left(\frac{5}{x} - 3g(x)dx\right)$

$= 5\ln|x| - 3\int g(x)dx$

$= 5\ln|x| - 3G(x) + c$

**d.** $\int \frac{f(x) + 3}{x}\, dx$

$= \int \frac{f(x)}{x} dx + 3\ln|x| + c$