Name ______________________________ Date ____________________

## Math 10250 Activity 25: Sketching Graphs (Section 4.3)

**GOAL:** To apply techniques from algebra and calculus to obtain a detailed sketch of the graph of a given function.

**Example 1** Sketch the graph of $f(x) = xe^{-x^2/2}$ by completing the steps below.

**a.** Find all $x$-intercepts and $y$-intercepts of the graph of $f(x)$ whenever possible.

$f(x) = 0 \Longrightarrow xe^{-\frac{x^2}{2}} = 0 \Longrightarrow x = 0$ *x-intercept*

$f(0) = 0 \Longrightarrow y = 0$ *y-intercept*

| $x$ | | 0 | |
|---|---|---|---|
| $f(x)$ | $-$ | 0 | $+$ |

**b.** Find coordinates of all critical points, vertical asymptotes, and places where $f(x)$ is undefined.

$f'(x) = (1 - x^2)e^{-\frac{x^2}{2}} = 0 \Longrightarrow x = -1, 1$ *critical points*

**c.** Determine where $f(x)$ is increasing and where it is decreasing.

| $x$ | | $-1$ | | $1$ | |
|---|---|---|---|---|---|
| $f'(x)$ | $-$ | 0 | $+$ | 0 | $-$ |
| $f(x)$ | $\searrow$ | *cp* *min* | $\nearrow$ | *cp* *max* | $\searrow$ |

**d.** Determine the concavity and coordinates of inflection points of $f(x)$. $(f''(x) = (x^3 - 3x)e^{-x^2/2})$

$$f''(x) = [-2x + (1 - x^2)(-x)]e^{-\frac{x^2}{2}}$$
$$= (x^3 - 3x)e^{-\frac{x^2}{2}}$$
$$f''(x) = 0 \Longrightarrow x(x^2 - 3) = 0$$
$$\Longrightarrow x(x - \sqrt{3})(x + \sqrt{3}) = 0$$
$$\Longrightarrow x = 0, \pm\sqrt{3}$$

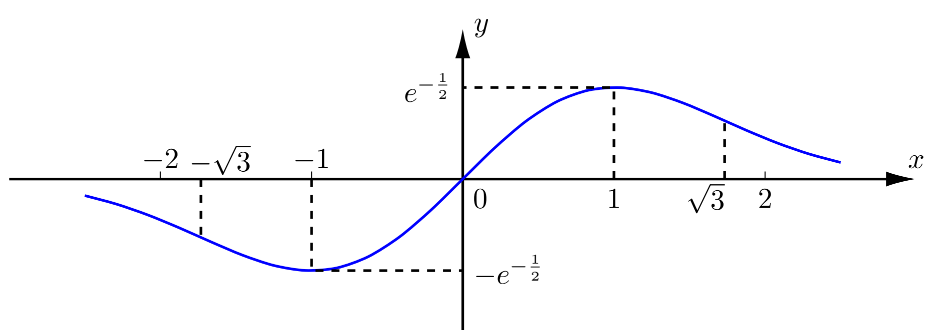
| $x$ | | $-\sqrt{3}$ | | $0$ | | $\sqrt{3}$ | |
|---|---|---|---|---|---|---|---|
| $f''(x)$ | $-$ | 0 | $+$ | 0 | $-$ | 0 | $+$ |
| $f(x)$ | $\frown$ | *ip* | $\smile$ | *ip* | $\frown$ | *ip* | $\smile$ |

**e.** Find all asymptotes and limit at infinity whenever applicable. Check for any symmetry.

$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty}(xe^{-\frac{x^2}{2}}) \approx \frac{\textit{Big}}{\textit{Huge}} = 0 \Longrightarrow y = 0$ *is horizontal asymptote .*

$f(-x) = -xe^{-\frac{x^2}{2}} = -f(x) \Longrightarrow$ *symmetric about origin.*

**f.** Sketch the graph below labeling all important features. Your picture should be large and clear.



**Example 2** Sketch the graph of $g(x) = \dfrac{x}{x^2-4}$ by completing the steps below.

**a.** Find all $x$-intercepts and $y$-intercepts of the graph of $g(x)$ whenever possible.

$g(x) = 0 \Longrightarrow x = 0$ *$x$-intercept* $x = \pm 2$ *vertical asymptote*

$g(0) = 0$ *$y$-intercept*

| $x$ | | $-2$ | | $0$ | | $2$ | |
|---|---|---|---|---|---|---|---|
| $g(x)$ | $-$ | v.a. | $+$ | $0$ | $-$ | v.a. | $+$ |

**b.** Find coordinates of all critical points, vertical asymptotes, and places where $g(x)$ is undefined.

$$g'(x) = \frac{1(x^2-4) - x(2x-0)}{(x^2-4)^2} = \frac{x^2-4-2x^2}{(x^2-4)^2} = -\frac{x^2+4}{(x^2-4)^2}$$

**c.** Determine where $g(x)$ is increasing and where it is decreasing.

| $x$ | | $-2$ | | $2$ | |
|---|---|---|---|---|---|
| $g'(x)$ | $-$ | | $-$ | | $-$ |
| $g(x)$ | $\searrow$ | v.a. | $\searrow$ | v.a. | $\searrow$ |

**d.** Determine the concavity and coordinates of inflection points of $g(x)$. $\left(g''(x) = \frac{(24+2x^2)x}{(x^2-4)^3} = \frac{24+2x^2}{(x^2-4)^2} \cdot \frac{x}{x^2-4}\right)$

$$g''(x) = -\frac{2x(x^2-4)^2 - 2(x^2-4)\cdot 2x(x^2+4)}{(x^2-4)^4}$$
$$= -\frac{2x^3 - 8x - 4x^3 - 16x}{(x^2-4)^3}$$
$$= \frac{2x^2+24}{(x^2-4)^2} \cdot \frac{x}{(x+2)(x-2)} = 0 \Longrightarrow x = 0$$

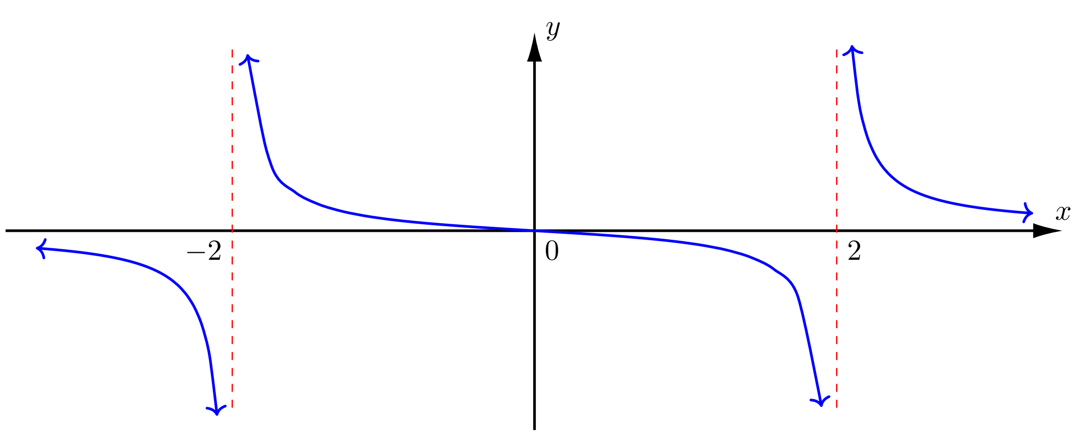
| $x$ | | $-2$ | | $0$ | | $2$ | |
|---|---|---|---|---|---|---|---|
| $g''(x)$ | $-$ | | $+$ | $0$ | $-$ | | $+$ |
| $g(x)$ | $\frown$ | v.a. concavity change | $\smile$ | ip | $\frown$ | v.a. concavity change | $\smile$ |

**e.** Find all asymptotes and limits at infinity whenever applicable. Check for any symmetry.

$$\lim_{x\to\pm\infty} g(x) = \lim_{x\to\pm\infty} \frac{x}{x^2-4} = 0 \Longrightarrow y = 0 \quad \textit{is horizontal asymptote.}$$

$$g(-x) = \frac{-x}{(-x)^2-4} = \frac{-x}{(x)^2-4} = -g(x) \Longrightarrow \quad \textit{symmetric about origin.}$$

**f.** Sketch the graph below labeling all important features. Your picture should be large and clear.




**Your Turn.** Sketch the graph of the solution to the Logistic Model for $r = 0.9$, $K = 10$, and $y_0 = 1$ or $y_0 = 6$:

$$y = f(t) = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}.$$