Name ______________________________ Date ____________________

Math 10250 Activity 6: Limits (Section 1.2 continued) and Continuity (Section 1.3)

GOAL: Understand behavior of functions at $\pm\infty$ and horizontal asymptotes. For rational functions the behavior at $\pm\infty$ is determined by the leading terms.

▶ Limits at infinity and horizontal asymptotes

- We say that $\lim_{x\to\infty} f(x) = L$ if $f(x) \approx L$ *when* x *is HUGE*

- We say that $\lim_{x\to-\infty} f(x) = L$ if $f(x) \approx L$ *when* x *is -HUGE*

- We say that $y = L$ is **horizontal asymptote** if
- $\lim_{x\to\infty} f(x) = L$ *and/or* $\lim_{x\to-\infty} f(x) = L$

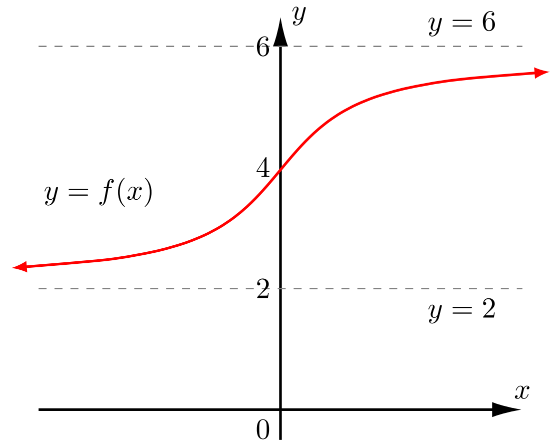


Figure 1

Example 1 For the function shown in Figure 1 find:

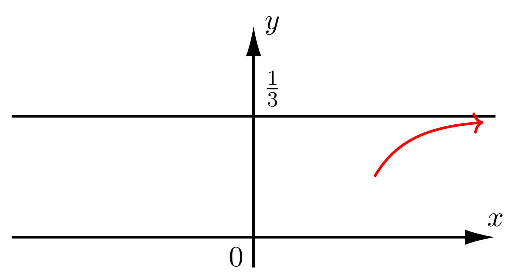
(i) $\lim_{x\to\infty} f(x) \stackrel{?}{=} 6$ and (ii) $\lim_{x\to-\infty} f(x) \stackrel{?}{=} 2$.

Also, find the horizontal asymptotes.

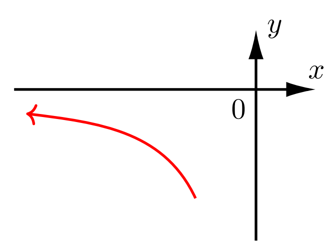
$\boxed{y = 6}$ and $\boxed{y = 2}$

Example 2

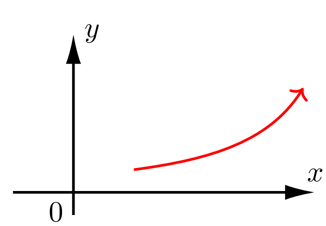
(i) $\lim_{x\to\infty} \dfrac{x^2+x}{3x^2+7} \stackrel{?}{=} \lim_{x\to\infty} \dfrac{\frac{x^2+x}{x^2}}{\frac{3x^2+7}{x^2}} = \lim_{x\to\infty} \dfrac{1+\frac{1}{x}}{3+\frac{7}{x^2}} = \dfrac{1+0}{3+0} = \boxed{\dfrac{1}{3}}$.



(ii) $\lim_{x\to-\infty} \dfrac{4x^3+7x^2}{x^4+2} \stackrel{?}{=} \lim_{x\to-\infty} \dfrac{\frac{4x^3+7x^2}{x^4}}{\frac{x^4+2}{x^4}} = \lim_{x\to-\infty} \dfrac{\frac{4}{x}+\frac{7}{x^2}}{1+\frac{2}{x^4}} = \dfrac{0+0}{1+0} = \boxed{0}$.



(iii) $\lim_{x\to\infty} \dfrac{x^3-2}{x^2+1} \stackrel{?}{=} \lim_{x\to\infty} \dfrac{\frac{x^3-2}{x^2}}{\frac{x^2+1}{x^2}} = \lim_{x\to\infty} \dfrac{x-\frac{2}{x^2}}{1+\frac{1}{x^2}} = \dfrac{\infty-0}{1+0} = \boxed{\infty}$.



Example 3 A company estimates that when it spends x million dollars to advertise its product, its annual revenue R, in millions of dollars, is modeled by the function $R(x) = 400 - \dfrac{800}{x+5}$.

(i) Compute $\lim_{x\to 0} R(x)$ and $\lim_{x\to\infty} R(x)$.

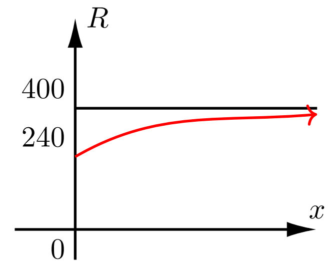
$\lim_{x\to 0} R(x) = 400 - \frac{800}{0+5} = 400 - 160 = 240$ *and* $\lim_{x\to\infty} R(x) = 400 - 0 = 400.$ $\lim_{x\to 0} R(x) = 240$ and $\lim_{x\to\infty} R(x) = 400$

(ii) If the company is currently spending 35 million on advertising, would you recommend increasing it to 40 million? To see this clearly, draw the graph of $R(x)$.

$$R(35) = 400 - \frac{800}{35+5} = 400 - \frac{800}{40} = 380$$

$$R(40) = 400 - \frac{800}{40+5} = 400 - \frac{800}{45} \approx 382.3$$

Increasing advertising from 35 *to* 40 *makes no sense.*



▶ **Idea of Continuity:** A function is continuous if you never have to lift your pencil while drawing its graph. The **discontinuities** are where you have to lift your pencil.

Definition of continuity

A function $f(x)$ is continuous at a point a **in** its **domain** if

1. $\lim_{x\to a} f(x)$ *exists* $\iff$ *no gaps*
2. $\lim_{x\to a} f(x) \stackrel{?}{=} f(a) \iff$ *no holes*

or $x \approx a \Longrightarrow f(x) \approx f(a)$

Example 4 Referring to the function f, whose graph is shown in Figure 2, find all the discontinuities of f in the interval $(-1.2, 7.2)$.

- $x = -1$ *since* $x \approx -1 \Longrightarrow f(x) \approx -2 \neq f(-1) = 1$ (*hole*)
- $x = 0$ *since* $x \approx 0 \Longrightarrow f(x) \not\approx f(0) = 1$ (*gap*)
- $x = 1$ *since* $x \approx 1 \Longrightarrow f(x) \approx 2 \neq f(1) = 2$ (*hole*)
- $x = 3$ *since* $x \approx 3 \Longrightarrow f(x) \approx 5 \neq f(3) = 4$ (*hole*)
- $x = 4$ $\cdots$
- $x = 5$
- $x = 6$

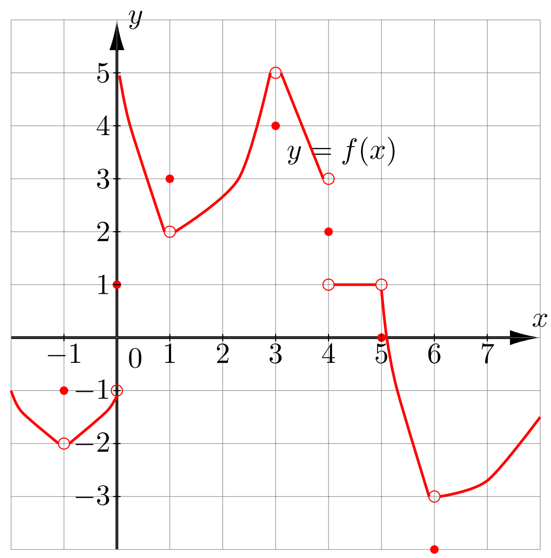


Figure 2