Name ________________________________ Date ____________________

Math 10250 Activity 2: Linear and Quadratic Functions (Sections 0.4 and 0.5)

GOAL: Understand the concept of slope for lines and linear functions and learn how to visualize quadratic functions by completing the square.

▶ A **linear function** is is defined by the formula:

$$f(x) = \underset{\text{slope}}{\underset{\uparrow}{m}}x + \underset{y\text{-intercept}}{\underset{\uparrow}{b}} \qquad \text{where } m \text{ and } b \text{ are given numbers.}$$

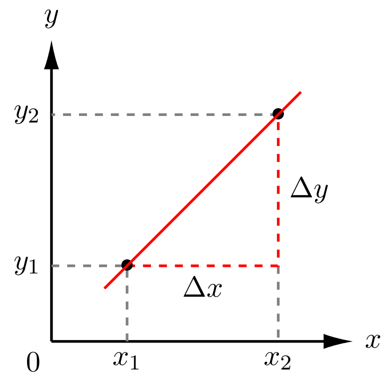


Figure 1

• Also, it is defined by a non-vertical line, like in Figure 1, having

$$\textbf{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \textit{the}\ \textbf{same!}\ \textit{for any}\ (x_1, y_1)\ \textit{and}\ (x_2, y_2)$$

Exercise 1 Find the slope of the line passing through $(-1, 1)$ and $(2, 7)$.

$$\textit{slope} = \frac{\Delta y}{\Delta x} = \frac{7-1}{2-(-1)} = \frac{6}{3} = 2.$$

• **Equation of line passing through a point** (x_1, y_1) **and with a given slope** m**:** If (x, y) is another point on the line then $\dfrac{y - y_1}{x - x_1} = m$. So we have the
point-slope form :

$$\boxed{y - y_1 = m(x - x_1)}$$

Exercise 2 Find the equation of the line through $(-1, 1)$ and with slope 2.

$$y - 1 = 2 \cdot (x - (-1)) \ \textit{ or } \ y - 1 = 2x + 2 \ \textit{ or } \ \boxed{y = 2x + 3}$$

Exercise 3 A small surf shop has fixed expenses of \$850 per month. Each surfboard costs \$100 to make and sells for \$550.

(a) Write the monthly **cost, revenue, and profit as functions** of the number x of surfboards made in a month.

Cost function $= C(x) \overset{?}{=} 100 \cdot x + 850$

Revenue function $= R(x) \overset{?}{=}$ *(price)* $\cdot$ *(quantity)* $= 550x$

Profit function $= P(x) \overset{?}{=}$ *Revenue* $-$ *Cost* $= 550x - (100x + 850) = 450x - 850$

(b) Find the break-even point. $0 = P(x) = 450x - 850 \Longrightarrow x = \frac{850}{450} \approx 2$

Ans. $x \approx 2$

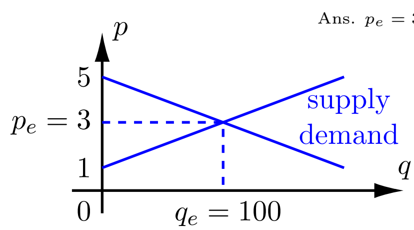
Exercise 4 The **demand curve** of bread in a bakery shop is $q = D(p) = -50(p - 5)$ and its **supply curve** is $q = S(p) = 50(p - 1)$, where the price p is in dollars and the quantity q is in loaves. Find the **equilibrium price** p_e and **equilibrium quantity** q_e.

Ans. $p_e = 3, q_e = 100$

Demand $=$ *supply* $\Longrightarrow -50(p - 5) = 50(p - 1)$

$\Longrightarrow p - 1 + p - 5 = 0 \Longrightarrow 2p = 6 \Longrightarrow \boxed{p_e = 3}$

$\Longrightarrow q_e = S(p_e) = 50(3 - 1) = 100 \Longrightarrow \boxed{q_e = 100}$



▶ A **quadratic function** is a function of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$, b and c are given numbers. It can always be written in the **informative** form $\boxed{f(x) = a(x-h)^2 + k}$, which is a **horizontal translation** by h and a **vertical translation** by k of the **simple parabola** $f(x) = ax^2$.

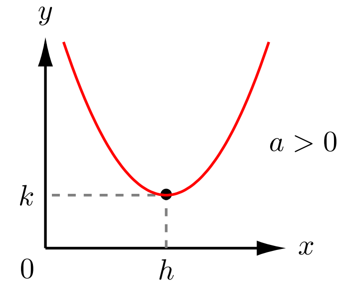


Figure 2

Exercise 5 Consider the quadratic function $f(x) = -x^2 + 6x - 5$.

(i) Complete the square to write it in the form $f(x) = a(x-h)^2 + k$.

$$\begin{aligned} f(x) &= -\left[x^2 + 6x\right] - 5 = -[x^2 - 6x] - 5 \\ &= -\left[x^2 - 2\cdot x \cdot 3\right] - 5 = -[x^2 - 2\cdot x\cdot 3 + 3^2 - 3^2] - 5 \\ &= -\left[(x-3)^2 - 9\right] - 5 = \boxed{-(x-3)^2 + 4} \end{aligned}$$

(ii) Use (i) to decide whether $f(x)$ has a minimum value or a maximum value and where it is taken.

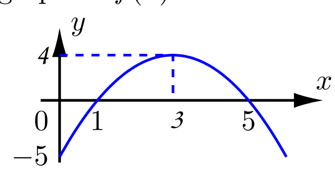
It has max value 4 *when* $x = 3$.

(iii) Use (i) to find the roots of $f(x)$.

$$-(x-3)^2 + 4 = 0 \iff (x-3)^2 - 2^2 = 0 \iff (x-3-2)(x-3+2) = 0 \iff (x-5)(x-1) = 0 \iff \boxed{x = 5, 1}$$

(iv) Determine the axis of symmetry and the y-intercept and sketch the graph of $f(x)$.

- $x = 3$ *is the axis of symmetry*
- $f(0) = -5$ *is the y-intercept*

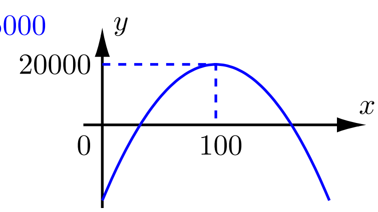


Exercise 6 A furniture company making oak desks has a fixed cost of \$5,000 per month and a cost per desk of \$500. Find how many desks per month it should produce to maximize its profit if the price is given by $p = 1000 - 2.5x$, where x denotes the number of oak desks produced by the company.

$$R(x) = x\cdot(1000 - 2.5x), \quad C(x) = 500x + 5000 \Longrightarrow$$

$$\begin{aligned} P(x) &= R(x) - C(x) = x\cdot(1000-2.5x) - (500x + 5000) = -2.5x^2 + 500x - 5000 \\ &= -2.5(x^2 - 200x) - 5000 = -2.5(x^2 - 2\cdot x\cdot 100 + 100^2 - 100^2) - 5000 \\ &= -2.5[(x-100)^2 - 10000] - 5000 = -2.5(x-100)^2 + 20000 \end{aligned}$$

$$\Longrightarrow \text{ max prof.} = 20000 \text{ when } \boxed{x = 100}$$



Exercise 7 Consider the quadratic $f(x) = x^2 - 5x + 4$.

(a) Find its zeros using the **quadratic** formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4\cdot 1\cdot 4}}{2\cdot 1} = \frac{5 \pm \sqrt{9}}{2} = \frac{5\pm 3}{2} \begin{cases} \frac{5+3}{2} = 4 \\ \frac{5-3}{2} = 1 \end{cases}$$

(b) Factor it. $f(x) = (x-1)(x-4)$

(c) Determine the sign of $f(x)$.

$+$	1	$-$	4	$+$
$x = 0$	⋮	$x = 2$	⋮	$x = 5$
$x - 1 < 0$	⋮	$x - 1 > 0$	⋮	$x - 1 > 0$
$x - 4 < 0$	⋮	$x - 4 < 0$	⋮	$x - 4 > 0$

check signs at $x = 5, 2$ *and* 0.