Name ______________________________ Date ____________________

Math 10250 Activity 1: Functions and Their Geometric Properties[1] **(Sections 0.2–0.3)**

GOAL: Understand the fundamental concept of function as a relation between variables expressed by a formula, a graph, or a table and use it to model change.

Q1: What is a variable? What is a function?

A1: *Variable models a changing quantity*
Function models a relationship between two variables, say x and y

- x *independent variable,*
- y *dependent variable,*
- $\boxed{\text{to an } x \text{ corresponding } \underline{\text{only}} \text{ one } y}$.

Example Assume that you have just deposited \$500 in your bank account at the ND Credit Union that pays annual interest 2% compounded daily, and you want to know what will be your amount at any future day. Use variables and functions to model it.

$t=0$ $\quad$ $t=1$ $\quad$ $t=2$ $\quad$ t *days* $\quad$ t

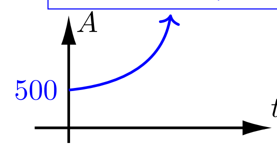
$A(0)=500,\quad A(1)=500+500\cdot(0.02)\cdot\frac{1}{365},\quad A(2)=A(1)+A(1)\cdot(0.02)\cdot\frac{1}{365},\cdots,\quad \boxed{A(t)=500\left(1+\frac{0.02}{365}\right)^{t}}$

$$=500\left(1+\frac{0.02}{365}\right)^{1}$$

$$=A(1)\left(1+\frac{0.02}{365}\right)$$

$$=500\left(1+\frac{0.02}{365}\right)^{2}$$

Variables:
$t =$ *time*
$A =$ *amount*



Exercise 1 Consider the function $f(x)=x^2+1$.

(a) Compute the following table of its values:

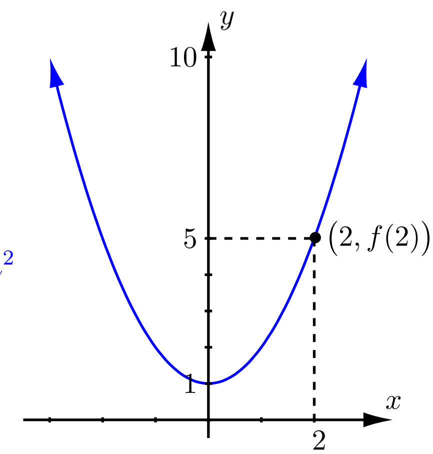
x	-3	-2	-1	0	1	2	3
$y=f(x)$	10	5	2	1	2	5	10

(b) Compute $f(1+h) = (1+h)^2+1 = 1+2h+h^2+1 = 2+2h+h^2$

(c) Find the (natural) domain of f. *all numbers*

(d) Find the range of f. *all numbers* ≥ 1 *since* $y = x^2+1 \geq 0+1$

(e) Sketch the graph of f. *It consists of all points* $(x, f(x))$.



Exercise 2 What is the natural domain of $f(x)=\dfrac{5}{x^2-9}$?

Not to violate the laws of arithmetic, we must have $x^2-9\neq 0$ *or* $(x-3)(x+3)\neq 0$ *or* $x\neq \pm 3$.

Exercise 3 Which of the curves below:

(a) is the graph of a function? (b) is the graph of a 1-1 function?

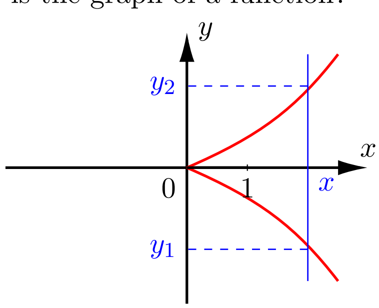


Figure 1: *Not a function*

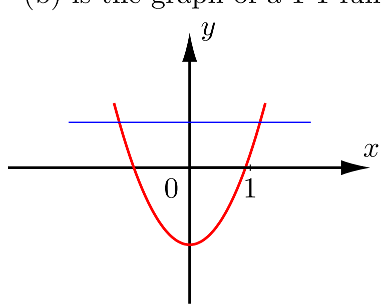


Figure 2: *a function, but not 1-1*

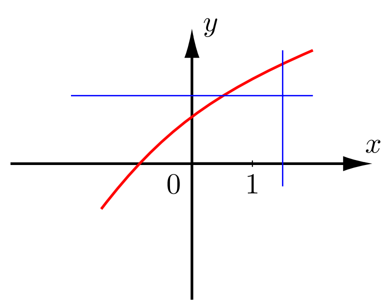


Figure 3: *a 1-1 function*

[1] Alex Himonas & Alan Howard: Calculus, Ideas and Applications, Wiley (2003).

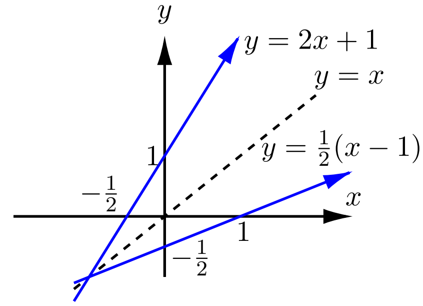
Exercise 4 Find the inverse of the function $y = f(x) = 2x + 1$ and sketch its graph.

Step 1. Solve $y = 2x + 1$ *for* x

We have $y - 1 = 2x$ *or* $x = \frac{1}{2}(y-1)$

Step 2. Interchange x *and* y

$\boxed{y = \frac{1}{2}(x-1)}$ *inverse*



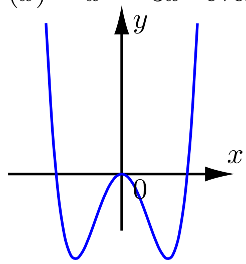
Exercise 5 For the function shown in Figure 2 determine where it is increasing and where it is decreasing.

- *Function in Figure 2 is increasing for* $x > 0$, *since* $0 < x_1 < x_2 \Longrightarrow f(x_1) < f(x_2)$.
- *Function in Figure 2 is decreasing for* $x < 0$, *since* $x_1 < x_2 < 0 \Longrightarrow f(x_1) > f(x_2)$.

Exercise 6 Is the function $f(x) = x^4 - 3x^2$ even or odd? What about $f(x) = x^3 + x$ and $f(x) = x^3 - x^2$?

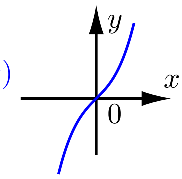
$$f(-x) = (-x)^4 - 3(-x)^2 = x^4 - 3x^2 = f(x)$$

So $f(x)$ *is even* $\Longrightarrow$ *graph symmetric w.r.t* y



$$f(-x) = (-x)^3 + (-x) = -x^3 - x = -f(x)$$

So $f(x)$ *is odd* $\Longrightarrow$ *graph symmetric w.r.t* 0



Exercise 7 For the the function $f(x)$, whose graph is shown in Figure 4, sketch the following vertical and horizontal translations

(a) $y = f(x) + 3$ (b) $y = f(x) - 1$ (c) $y = f(x-1)$ (d) $y = f(x+2)$ (e) $y = f(x+2) + 3$

on the same system of Cartesian plane.

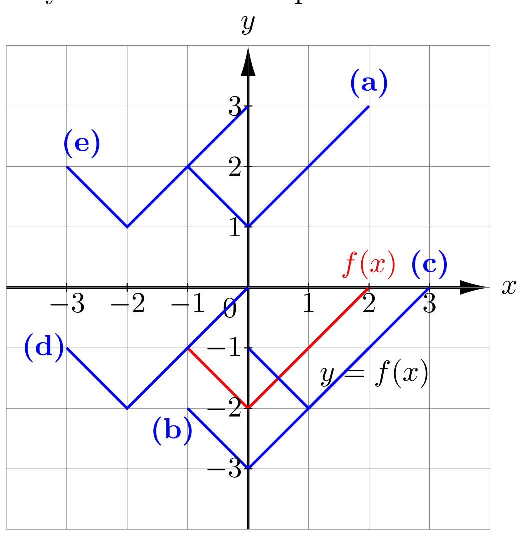


Figure 4

(a) vertical translation (v.t.) of graph of $f(x)$, *up* 3 *units*

(b) v.t. of graph of $f(x)$, *down* 1 *unit*

(c) horizontal translation (h.t.) of graph of $f(x)$, *right by* 1 *unit*

(d) h.t. of graph $f(x)$, *left* 2 *units*

(e) h.t. of graph of $f(x)$, *left* 2 *units and v.t., up* 3 *units.*

Exercise 8 Sketch the graph of the functions $y = x^2$ and $y = (x-3)^2 + 1$.