

Math 10250 Activity 35: Computing Definite Integrals, Areas, Averages, & Numerics (5.6-6.1)

GOAL: Apply the method of integration by parts with the Fundamental Theorem of Calculus to compute Integrals, Areas, Averages. Also, estimate integrals via numerical methods. Finally, compute Consumer and Producer Surplus for given demand and supply curves.

► Integration by parts in definite integrals

$$\int_a^b u \, dv =$$

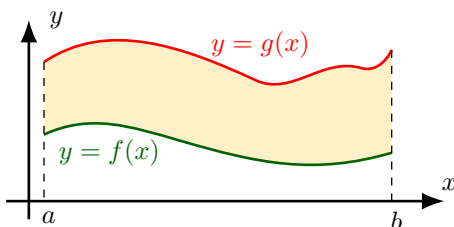
Example 1

(a) $\int_0^1 t e^{t/2} \, dt =$

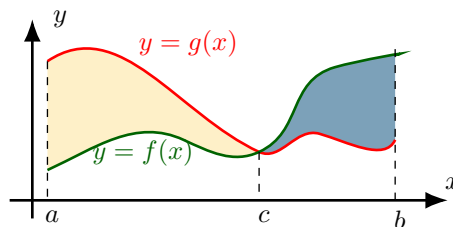
(b) $\int_1^2 x \ln(x) \, dx =$

► The area between two curves

Consider the following region:



Now consider:



Area between f and $g = \int_a^b$

Area between f and $g = \int_a^c [g(x) - f(x)] \, dx + \int_c^b [f(x) - g(x)] \, dx.$

Example 2 Find the intersection points of $f(x) = 1 - x^2$ and $g(x) = x + 1$. Then find the area between the graphs over the interval $-1 \leq x \leq 1$.

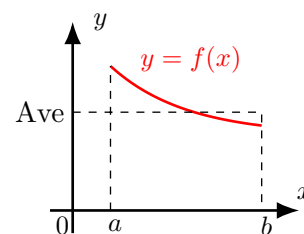
► Average values of continuous quantities

Q1: What is the average value of 3, 5 and 7?

A1: _____

Q2: What is the average value of $f(x)$ on $[a, b]$?

A2: Average value of f over $[a, b] = \frac{\int_a^b$ _____



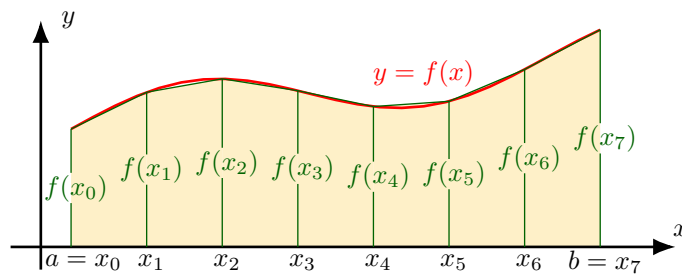
Example 3 Find the average value of $f(x) = x^5$ over the interval $[-1, 2]$.

Example 4 Estimate $\int_0^1 e^{\sqrt{x}} dx$

(a) using the mid-point with $n = 4$.

(b) using trapezoidal rule with $n = 4$. (See description below.)

► **The trapezoidal rule:** To estimate $\int_a^b f(x) dx$, we can use trapezoids instead of rectangles. Recall that the area of a trapezoid = $\frac{1}{2}$ (sum of the two parallel sides)·(height).

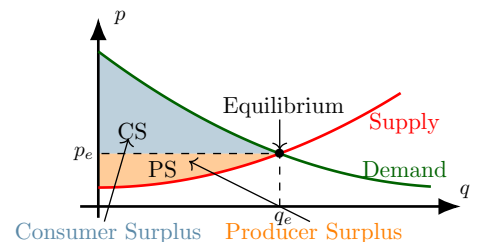


$$\int_a^b f(x) dx \approx [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)] \cdot \frac{\Delta x}{2} \leftarrow \text{trapezoidal rule}$$

► **Consumer/Producer Surplus:** In a free market economy, both the consumers and producers benefit by buying/selling at the equilibrium price. It can be shown (see Section 6.1) that:

$$\text{Consumer Surplus (CS)} = \int$$

$$\text{Producer Surplus (PS)} = \int$$



Example 5 Find the equilibrium quantity and price, the consumer surplus and the producer surplus for the demand curve $D(q) = \frac{110}{q+4}$ and the supply curve $S(q) = q + 5$. (Ans: $q_e = 6$, $p_e = 11$, CS 34.79, PS = 18)