Name ________________________________ Date ____________________

Math 10250 Activity 34: The Fundamental Theorem of Calculus (Section 5.5)

GOAL: Understand the Fundamental Theorem of Calculus (FTC) and use it to compute integrals.

Q1: What is the connection between $\int_a^b f(x)\,dx$ and $\int f(x)\,dx$?

↑ definite integral ↑ indefinite integral

A2:

Fundamental Theorem of Calculus

IF (1) $f(x)$ is continuous on $[a, b]$ and (2) $F(x)$ is an antiderivative of $f(x)$; i.e., $F'(x) = f(x)$,

THEN $\int_a^b f(x)dx =$ ____________ ;i.e., $\int_a^b F'(x)dx =$ ________________________

Example 1 Compute the following definite integrals:

(a) $\int_1^2 (x^2 + 3)\,dx$

(b) $\int_{-2}^{-1} (e^{2x} + \frac{2}{x})\,dx$

Example 2 Sketch the graph of $f(x) = 2e^x$ from $a = -1$ to $b = 2$ and use the fundamental theorem of calculus to find the area under the graph.

$\int_{-1}^{2} 2e^x\,dx =$

▶ Physical interpretations of the Fundamental Theorem of Calculus

** **Total change** of a certain quantity is expressed as the **definite integral** of its rate of change.**

- **From velocity** v **to displacement** s:

Displacement between times a and $b = s(b) - s(a) = \int_a^b$ ______ dt.

↑ change of position between times a and b

Example 3 An object is falling vertically downward, and its velocity (in feet per second) is given by $v = -32t - 20$. Write a definite integral that gives the change in height in the first 3 seconds.

Similary, the following are true.

- **From acceleration** a **to velocity** v:

Change in velocity between times a and $b = v(b) - v(a) = \int_a^b$ ______ dt.

• **From rate of growth $r(t)$ to total growth $g(t)$:**

$$\boxed{\text{Total growth between times } a \text{ and } b = g(b) - g(a) = \int_a^b \underline{\hspace{2cm}}\, dt.}$$

▶ From marginal function to total function

- The additional profit resulting in increasing production from a units to b units is given by

$$\boxed{\text{Total change in profit } \stackrel{?}{=} \qquad\qquad \stackrel{?}{=} \qquad\qquad = \int_a^b MP(x)\, dx.}$$

- The extra revenue resulting from increasing production from a units to b units is given by

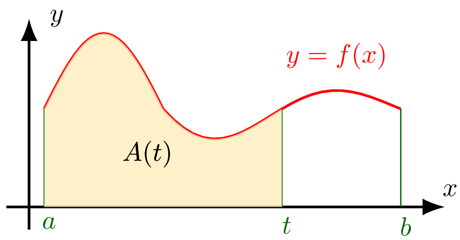
$$\boxed{\text{Total change in revenue } \stackrel{?}{=} \qquad\qquad \stackrel{?}{=} \qquad\qquad \stackrel{?}{=} \qquad\qquad .}$$

Example 4 Suppose the marginal cost involved in producing x units of a certain product is given by the function

$$MC(x) = 2x + 1000 \text{ when } x \geq 50.$$

Determine the increase in cost if production is increased from 50 to 80.

▶ The area as an antiderivative



Let $\mathrm{A}(t) = \int_a^t f(x)\, dx$ for $a \leq t \leq b$.

If $F(t)$ is an antiderivative of $f(t)$, what is the relation between $A(t)$ and $F(t)$? (Hint: Fundamental Theorem of Calculus)

<u>Conclusion</u>: $A(t)$ is *also* an antiderivative of $f(t)$, i.e.,

Theorem 5.5.2

IF $f(x)$ is continuous on $[a, b]$ THEN $\dfrac{d}{dt}\int_a^t f(x)dx \stackrel{?}{=}$

Example 5 $\dfrac{d}{dt}\int_1^t (1 + \ln x)^2\, dx \stackrel{?}{=}$.

▶ Substitution in definite integrals

$$\boxed{\int_a^b f(g(x))g'(x)\, dx \stackrel{u=g(x)}{=} \qquad\qquad}$$

Example 6

(a) $\int_4^5 x\sqrt{x^2 - 16}\, dx \stackrel{?}{=}$

(b) $\int_0^1 xe^{x^2}\, dx \stackrel{?}{=}$