

### Math 10250 Activity 29: The Indefinite Integral (Section 5.1)

**GOAL:** If we are given the derivative  $f'(x)$ , we want to be able to find the function  $f(x)$ .

► **Antiderivatives** (Reversing differentiation)

**Definition:**  $F'(x) = f(x)$  means that  $F(x)$  is an **antiderivative** of  $f(x)$ .

**Example 1** Find all the antiderivatives of the indicated function  $f(x)$ . That is, find all the functions  $F(x)$  so that when we take their derivative, we get  $f(x)$ . In each case, sketch three of them on the same set of axes.

$$f(x) = 0$$

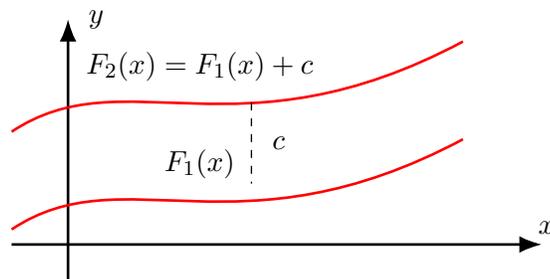
$$f(x) = 2x$$

From Example 1, we see that

**Theorem** If  $F_1(x)$  and  $F_2(x)$  are antiderivatives of the same function throughout an interval, then they differ by a constant  $c$  over that interval; that is, for  $a < x < b$

$$F_1'(x) = F_2'(x) \Rightarrow$$

for some number  $c$ .



**Q1:** How do we denote all antiderivatives of  $f(x)$ ?

**A1:** If  $F(x)$  is an antiderivative of  $f(x)$ ; that is,  $F'(x) = f(x)$ . Then we may write

$$\int f(x)dx = \underline{\hspace{10em}}$$

We call  $\int f(x)dx$  the **indefinite integral**.

**Example 2** Let  $f(x) = (5x - 1)^3$  and  $F(x) = A(5x - 1)^4$ .

a. Find the value of the constant  $A$  that makes  $F(x)$  an antiderivative of  $f(x)$ .

b. Write your result in Part (a) in terms of the indefinite integral.

**Example 3** Referring to Example 1, find the indefinite integral of  $f(x) = 2x$ .

**Example 4** If  $k \neq 0$ , compute  $\frac{d}{dx}(e^{kx})$  then write down  $\int e^{kx} dx$ . (Use the fact:  $(c \cdot f(x))' = c \cdot f'(x)$ )

**Example 5** If  $k \neq -1$ , compute  $\frac{d}{dx}(x^{k+1})$  then write down  $\int x^k dx$ .

► **Basic indefinite integral formulas**

• For any constant  $m$ :  $\int m dx \stackrel{?}{=} \boxed{\phantom{m x + c}}$ . For Example:  $\int 100 dx \stackrel{?}{=} \phantom{\int 100 dx}$

• Power Rule when  $k \neq -1$ :  $\int x^k dx \stackrel{?}{=} \boxed{\phantom{x^{k+1} + c}}$ . For Example:  $\int x^9 dx \stackrel{?}{=} \phantom{\int x^9 dx}$

• Power Rule when  $k = -1$ :  $\int \frac{1}{x} dx = \boxed{\phantom{\ln|x| + c}}$ .

• Exponential Rule:  $\int e^{kx} dx = \boxed{\phantom{\frac{1}{k} e^{kx} + c}}, k \neq 0$  For Example:  $\int e^{0.1x} dx \stackrel{?}{=} \phantom{\int e^{0.1x} dx}$

• Constant Multiple Rule:  $\int kf(x) dx = k \int f(x) dx, \text{ any } k$  For Example:  $\int \frac{8}{x} dx \stackrel{?}{=} \phantom{\int \frac{8}{x} dx}$

• Sum Rule:  $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx.$

**Example 6** Find each of the following indefinite integrals. Check your answer by differentiation.

a.  $\int \left( x^7 - 2x^{-4} + \frac{3}{x} + e^{2x} \right) dx$

b.  $\int \frac{3x - 10x^2 + \sqrt{x}}{x^3} dx$

**Example 7** Given that  $\int f(x) dx = F(x) + c$  and  $G'(x) = g(x)$ . Find each of the following indefinite integrals in terms of  $F(x)$ ,  $G(x)$ , and other known functions whenever possible. If not possible, state so.

a.  $\int [2f(x) + 3x] dx$

c.  $\int \frac{5 - 3x \cdot g(x)}{x} dx$

b.  $\int f(x) \cdot g(x) dx$

d.  $\int \frac{f(x) + 3}{x} dx$