

Math 10250 Activity 25: Sketching Graphs (Section 4.3)

GOAL: To apply techniques from algebra and calculus to obtain a detailed sketch of the graph of a given function.

Example 1 Sketch the graph of $f(x) = xe^{-x^2/2}$ by completing the steps below.

- a. Find all x -intercepts and y -intercepts of the graph of $f(x)$ whenever possible.

- b. Find coordinates of all critical points, vertical asymptotes, and places where $f(x)$ is undefined.

- c. Determine where $f(x)$ is increasing and where it is decreasing.

- d. Determine the concavity and coordinates of inflection points of $f(x)$. $(f''(x) = (x^3 - 3x)e^{-x^2/2})$

- e. Find all asymptotes and limit at infinity whenever applicable. Check for any symmetry.

- f. Sketch the graph below labeling all important features. Your picture should be large and clear.

Example 2 Sketch the graph of $g(x) = \frac{x}{x^2 - 4}$ by completing the steps below.

a. Find all x -intercepts and y -intercepts of the graph of $g(x)$ whenever possible.

b. Find coordinates of all critical points, vertical asymptotes, and places where $g(x)$ is undefined.

c. Determine where $g(x)$ is increasing and where it is decreasing.

d. Determine the concavity and coordinates of inflection points of $g(x)$.

$$\left(g''(x) = \frac{(24 + 2x^2)x}{(x^2 - 4)^3} = \frac{24 + 2x^2}{(x^2 - 4)^2} \cdot \frac{x}{x^2 - 4} \right)$$

e. Find all asymptotes and limits at infinity whenever applicable. Check for any symmetry.

f. Sketch the graph below labeling all important features. Your picture should be large and clear.