Name _____ Date _____

Math 10250 Activity 18: The Product and Quotient Rules (Section 3.6)

GOAL: To learn how to compute the derivatives of a product and a quotient of two functions.

▶ **The Product Rule:** $\frac{d}{dx}[f(x) \cdot g(x)] =$ _____

Note: $\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[g(x) \cdot f(x)]$.

Example 1 Use the product rule to find the derivatives:

(a) $\frac{d}{dx}[x^2(3x^3 - x)]$

(b) $\frac{d}{dx}[e^{-2x} \ln x]$

▶ **The Quotient Rule:** $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$ _____

In general, $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] \neq \frac{d}{dx}\left[\frac{g(x)}{f(x)}\right]$.

Example 2 Find the equation of the tangent line to the graph of $y = f(x) = \frac{x}{x^2+1}$ at the point $x = 2$.

Example 3 Use the appropriate differentiation rules that you have learned so far to find the derivatives below. Some algebra may be helpful.

(a) $\frac{d}{dx}\left(\frac{x^2 + x - 3}{100}\right)$

(b) $\frac{d}{dx}\left(\frac{x + e^x}{e^x}\right)$

(c) $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right)$

(d) $\frac{d}{dx}\left(\frac{x^2 + x - 3}{x^{10}}\right)$

Example 4 Suppose the demand for a certain product is given by $q = f(p)$, where p is the price per unit and q is the number sold. The revenue is given by $R = pq$.

(a) If $f(300) = 20,000$ and $f'(300) = -30$, find dR/dp when $p = 300$.

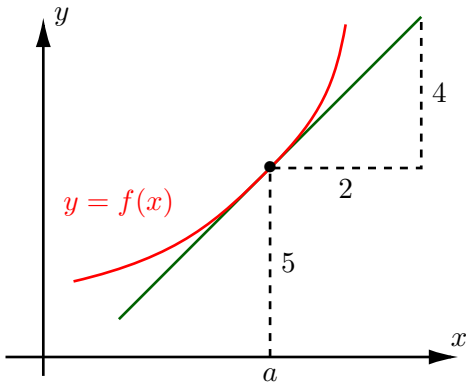
(b) If the product is currently selling for \$300 per unit, should the company increase or decrease the price in order to raise the revenue?

Example 5 For what x does the graph $y = xe^x$ have slope zero? Ans: $x = -1$

Example 6 Find the equation of the tangent line to the graph of $y = \dfrac{1 - \ln x}{1 + \ln x}$ at $x = 1$. Ans: $y = -2x + 3$

Example 7 Let $p(x) = f(x)g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$. Using the graph of $f(x)$ and $g(x)$ below find

(a) $p'(a)$



(b) $q'(a)$

Ans: $p'(a) = 10$ and $q'(a) = 0.3$