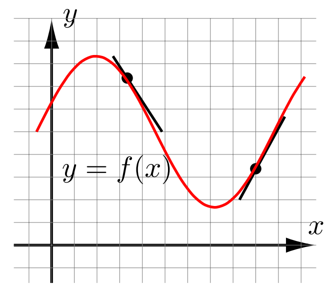
Name ______________________________ Date ____________________

Math 10250 Activity 12: The Slope of a Graph (Section 3.1)

GOAL: Understand the fundamental concept of the slope to a curve using limits and slope of lines. Also realize that slope to a curve is the same as instantaneous rate of change.

The **slope** at the point $(a, f(a))$ on the graph of $y = f(x)$ is the **slope of the tangent line** to the graph at $(a, f(a))$. We need two key concepts to find the slope at each point on the graph of $y = f(x)$:

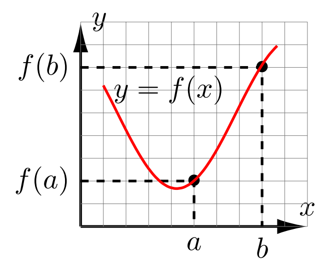


- Slope of line (Already done!)
- Limits (Already done!)
- Average rate of change (To be done).

▶ Average Rate of Change

Definition: The average rate of change of $f(x)$ over the interval $[a, b]$ is ____________________.

Graphical Interpretation: Use the graph here to explain the graphical meaning of average rate of change of $f(x)$ over an interval [a,b].

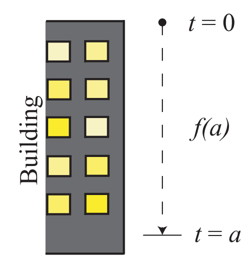


Linear Model:

Example 1 Find the average rate of change of $f(x) = 2x + 1$ at $x = 1$.

Nonlinear Model of Galileo: It can be shown experimentally that the distance travelled by a stone released at rest from the top of a building is given by $f(t) = 16t^2$.

Q1: Compute the following:



(a) Average speed over $1 \le t \le 3 = \frac{\text{Change in distance}}{\text{Change in time}} =$

(b) Average speed over $1 \le t \le 1 + h =$ ______________ $=$

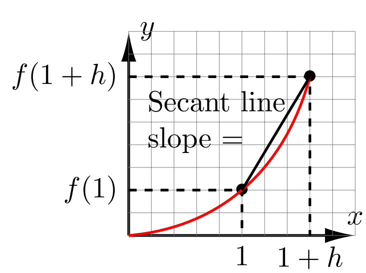
(c) Complete the table:

h	-0.01	-0.001	0	0.001	0.01
$\frac{f(1+h)-f(1)}{h}$			?		

Q2: What is the value of $L = \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h}$? What physical quantity does L represents?

Remark: We also call the value L the instantaneous rate of change of $f(t) = 16t^2$ at $t = 1$.

Use the graph here to give a graphical interpretation of the value of $L = \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h}$.



▶ **Instantaneous Rate of Change**

Definition: The instantaneous rate of change of $f(x)$ at $x = a$ is the value of the limit

$$\lim_{h \to 0} \left(\qquad \qquad \qquad \qquad \right)$$

Remark: Graphically, the instantaneous rate of change of $f(x)$ at $x = a$ is the **slope** of the **tangent line** to the curve $y = f(x)$ at the point $(a, f(a))$.

Example 2 Consider the function $f(x) = x^2 - 5x + 4$.

(i) Find the instantaneous rate of change of $f(x)$ at $x = 3$ using limits.

Step 1: Find (form) the average rate of change from 3 to $3 + h$ (or the slope of the secant line joining $(3, f(3))$ and $(3 + h, f(3 + h))$.

Step 2: Let $h \to 0$ in the slope of the secant line.

(ii) What is the equation of tangent line to the graph of $y = f(x)$ at $x = 3$?

(iii) Using the steps in (i), find an expression for the slope of the graph $y = f(x)$ at any given x.

Example 3 Using limits, find a formula for the instantaneous rate of change and slope of the following **important functions**:

- $f(x) = x^2$, for any x. Ans. $2x$
- $f(x) = \sqrt{x}$, for any $x > 0$. Ans. $\frac{1}{2\sqrt{x}}$