

Math 10250 Activity 10: Logarithmic Functions (Section 2.3)

GOAL: Learn logarithmic functions as inverses of exponential functions and use them to model various interesting situations, like intensity of earthquakes, noise level, and acidity of beer.

Q1: What “undoes” the exponential function $f(x) = b^x$? (For example, if $f(x) = 2^x$ then $3 \xrightarrow{f} 2^3 = 8$.)

A1: The **logarithmic function with base b** , denoted \log_b . (If $g(x) = \log_2 x$ then $8 \xrightarrow{g} \log_2 8 = 3$.)

Definition: \log_b (for $b > 0, b \neq 1$) is defined by

$$\log_b x = y \iff \dots$$

Example 1 Express the following logarithms as an integer or fraction without using a calculator.

(a) $\log_3 9$

(b) $\log_{(0.1)} 1000$

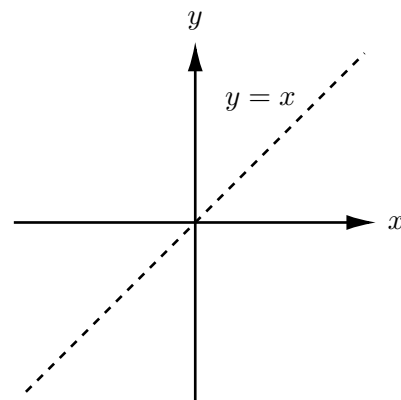
► **The graph of $\log_b x$ for $b > 1$**

As an example, first graph $y = 2^x$ and obtain the graph of $y = \log_2 x$.

Properties of logarithmic functions

- $\log_b 1 \stackrel{?}{=}$
- domain $\stackrel{?}{=}$ and range $\stackrel{?}{=}$
- It's continuous and increasing.
- $\lim_{x \rightarrow \infty} \log_b x \stackrel{?}{=}$ and $\lim_{x \rightarrow 0^+} \log_b x \stackrel{?}{=}$

Note: The most common choices for b are 10, e and 2.



► **The laws of logarithms.** (*Reversing the laws of exponents*) Let $s, t > 0$. Then

(1) $\log_b(st) \stackrel{?}{=}$;e.g., $\log_2(3 \cdot 8) \stackrel{?}{=}$

(2) $\log_b\left(\frac{s}{t}\right) \stackrel{?}{=}$;e.g., $\log_2\left(\frac{3}{8}\right) \stackrel{?}{=}$

(3) $\log_b(t^r) \stackrel{?}{=}$ for any number r ;e.g., $\log_2(3^7) \stackrel{?}{=}$

(4) $\log_b 1 \stackrel{?}{=}$

(5) $\log_b\left(\frac{1}{t}\right) \stackrel{?}{=}$;e.g., $\log_2\left(\frac{1}{8}\right) \stackrel{?}{=}$

Q2: Can you explain property (1)?

A2:

Example 2 Use the approximation $\log_{10} 0.5 \approx -0.301$ to estimate $\log_{10} 20$.

Example 3 Use the approximation $\log_2 3 \approx 1.585$ and $\log_2 5 \approx 2.322$ to estimate $\log_2 45$.

Example 4 Suppose A and b are positive numbers with $\log_3 A = b$. Write $\log_3 \left(\frac{3}{\sqrt[3]{A}} \right)$ in terms of b .

Example 5 A bank teller claims that a saving account with principal of \$1000 earning interest at a annual rate of 1.3%, compounded weekly, after T years would at least double. What is the smallest possible T in whole years?

Ans. 54 years.

► Logarithms with base 10

Logarithms with base 10, called **common logarithms**, are used in many well-known applications.

1 The Richter scale

$$\text{Richter value} = \log_{10} \left(\frac{x}{A} \right),$$

where A is the amplitude of the seismic wave of a reference earthquake and x is the amplitude of the seismic wave of the earthquake in question.

Example 6 One of the worst earthquakes in history occurred in Tokyo and registered 8.3 on the Richter scale. A more recent earthquake in California in 1989 registered 7.2. How much more severe was the earthquake in Tokyo in terms of the amplitude of its seismic wave?

Ans. $10^{1.1}$ larger.

2 The decibel scale

$$\text{Noise level in decibels} = 10 \log_{10} \left(\frac{x}{I} \right),$$

where I is the amplitude of a minimal audible sound wave and x is the amplitude of another sound wave. **Read Text Example 2.3.3 (p. 141).**

3 The pH scale

$$\text{pH value} = -\log_{10}[\text{H}^+],$$

where $[\text{H}^+]$ is the concentration of hydrogen ions in a solution. **Read Text Example 2.3.4 (p. 142).**