Name _____ Date _____

Math 10250 Activity 6: Limits (Section 1.2 continued) and Continuity (Section 1.3)

GOAL: Understand behavior of functions at $\pm\infty$ and horizontal asymptotes. For rational functions the behavior at $\pm\infty$ is determined by the leading terms.

▶ Limits at infinity and horizontal asymptotes

- We say that $\lim_{x\to\infty} f(x) = L$ if $\cdots$

- We say that $\lim_{x\to-\infty} f(x) = L$ if $\cdots$

- We say that $y = L$ is **horizontal asymptote** if

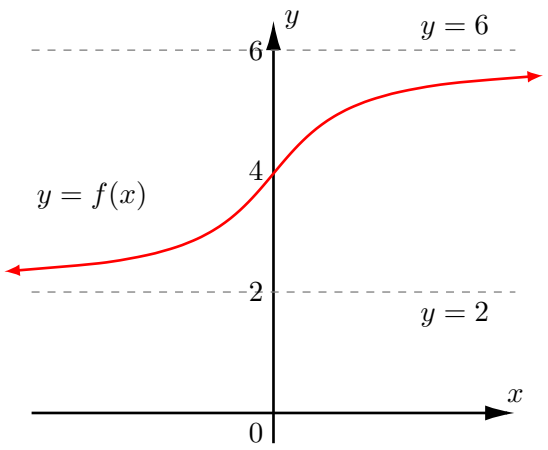

Figure 1

Example 1 For the function shown in Figure 1 find:

(i) $\lim_{x\to\infty} f(x) \stackrel{?}{=}$ and (ii) $\lim_{x\to-\infty} f(x) \stackrel{?}{=}$.

Also, find the horizontal asymptotes.

Example 2

(i) $\lim_{x\to\infty} \dfrac{x^2+x}{3x^2+7} \stackrel{?}{=}$

(ii) $\lim_{x\to-\infty} \dfrac{4x^3+7x^2}{x^4+2} \stackrel{?}{=}$

(iii) $\lim_{x\to\infty} \dfrac{x^3-2}{x^2+1} \stackrel{?}{=}$

Example 3 A company estimates that when it spends x million dollars to advertise its product, its annual revenue R, in millions of dollars, is modeled by the function $R(x) = 400 - \dfrac{800}{x+5}$.

(i) Compute $\lim_{x\to 0} R(x)$ and $\lim_{x\to\infty} R(x)$.

$\lim_{x\to 0} R(x) = 240$ and $\lim_{x\to\infty} R(x) = 400$

(ii) If the company is currently spending 35 million on advertising, would you recommend increasing it to 40 million? To see this clearly, draw the graph of $R(x)$.

▶ **Idea of Continuity:** A function is continuous if you never have to lift your pencil while drawing its graph. The **discontinuities** are where you have to lift your pencil.

Definition of continuity

A function $f(x)$ is continuous at a point a **in** its **domain** if
1. $\lim_{x\to a} f(x)$ _____
2. $\lim_{x\to a} f(x) \stackrel{?}{=}$

Example 4 Referring to the function f, whose graph is shown in Figure 2, find all the discontinuities of f in the interval $(-1.2, 7.2)$.

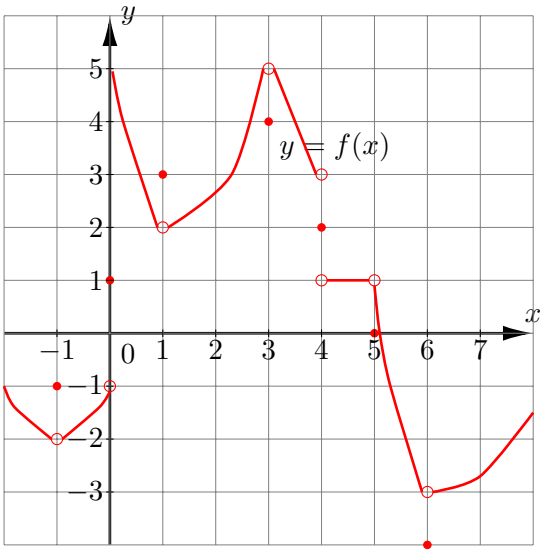

Figure 2