Name _____ Date _____

Math 10250 Activity 5: One-sided and Infinite Limits (Sec. 1.1 continued & Sec. 1.2)

GOAL: To learn about the limit of a function $f(x)$ as x approaches a number a from one side (left or right), get an understanding of infinite limits and relate them to vertical asymptotes.

▶ One-sided limits

Example 1 For the function $y = f(x)$ whose graph is shown in Figure 1, find (by visual inspection) the indicated **one-sided limits** (if they exist) and determine whether the limit of $f(x)$ exists at the given values of x.

(i) $\lim_{x\to -1^-} f(x) \stackrel{?}{=}$ ↑ Left-hand limit $\lim_{x\to -1^+} f(x) \stackrel{?}{=}$ ↑ Right-hand limit $f(-1) \stackrel{?}{=}$

(ii) $x = 0$

(iii) $x = 1$

(iv) $x = 3$

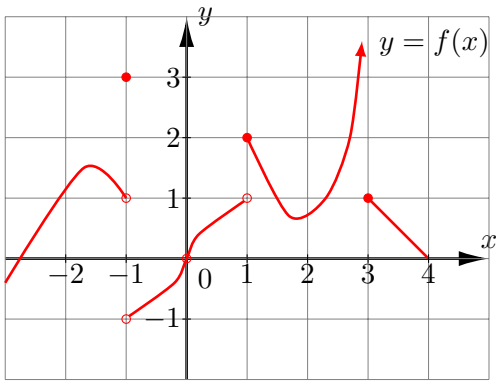

Figure 1

- **Definition of one-sided limits.**
  - ⋆ $\lim_{x\longrightarrow a^+} f(x) = L \Longleftrightarrow x \underset{x>a}{\approx} a \Longrightarrow f(x) \approx L$
  - ⋆ $\lim_{x\longrightarrow a^-} f(x) = L \Longleftrightarrow x \underset{x<a}{\approx} a \Longrightarrow f(x) \approx L$
- **Fact:** $\lim_{x\to a} f(x) = L$ if and only if
- **Rules of one-sided limits**. They are the **same** as the ones for usual (double-sided) limits.

Example 2 Find $\lim_{t\to 1^+} \dfrac{t^2-1}{\sqrt{t-1}}$.

Example 3 If $f(x)$ is the function in Example 1 and $g(x) = 8x - 1$, then find the following one-sided limits:

(i) $\lim_{x\to 1^+} [f(x)\cdot g(x)] \stackrel{?}{=}$

(ii) $\lim_{x\to 1^-} \dfrac{f(x)}{g(x)} \stackrel{?}{=}$

▶ Explain the meaning of the **infinite limits:**

- $\lim_{x \to a} f(x) = \infty$
- $\lim_{x \to a} f(x) = -\infty$
- $\lim_{x \to a^+} f(x) = \infty$ (or $-\infty$)
- $\lim_{x \to a^-} f(x) = \infty$ (or $-\infty$)

Example 4 For the function whose graph is shown in Figure 2 determine its limiting behavior as x approaches each of the points:

(i) $x = -2$ (ii) $x = 0$ (iii) $x = 2$ (iv) $x = 4$,

and find its **vertical asymptotes** (v.a.).

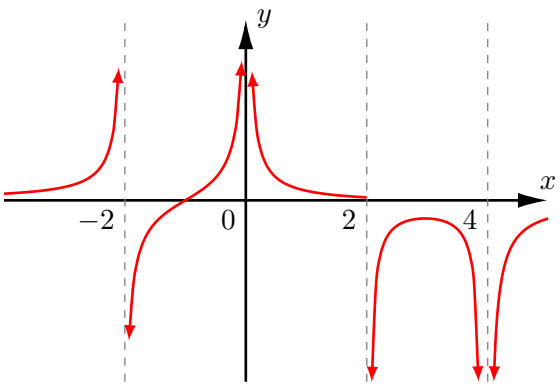

Figure 2

Example 5 $\lim_{x \to 3} \dfrac{1}{(x-3)^2} \stackrel{?}{=}$

Ans. ∞

Example 6 $\lim_{x \to 3} \dfrac{x}{x^2 - 9} \stackrel{?}{=}$

Hint. Check both left and right hand limits.

Ans. DNE