Name _____ Date _____

Math 10250 Activity 4: Limits (Section 1.1)

GOAL: To obtain an intuitive understanding of the fundamental concept of limit and learn rules for computing it.

Q1: Using your intuition, how would you interpret the statement: The function $f(x) = \dfrac{x^2 - 2x - 3}{x - 3}$ has limit 4 as x goes to 3?

A1: Natural domain of f: _____.

Since f is not defined at $x = 3$, let's look at how f behaves near $x = 3$. To do this, we make a table of values like this:

x	2.97	2.98	2.99	3	3.01	3.02	3.03
$f(x) = \dfrac{x^2 - 2x - 3}{x - 3}$				?			

Pattern: $f(x)$ gets close to ____ as x gets close to 3.

To make this more precise we need the help of algebra. So, let us factor the numerator of f:

$f(x) = \dfrac{x^2 - 2x - 3}{x - 3} =$

Sketch of $y = f(x)$:

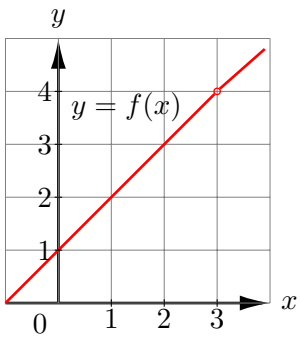

Figure 1

Now, we are confident to claim that the limit of $f(x)$ as x goes to 3 is 4.

We write this as: $\lim\limits_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3} = 4.$

Q2: Give an **Informal Definition of Limit**.

A2:

Exercise 1 The graph of a function f is shown in Figure 2. By visually inspecting the graph, find each of the following limits if it exists. If the limit does not exist, explain why.

(i) $\lim_{x\to 4} f(x) \stackrel{?}{=}$

(ii) $\lim_{x\to -1} f(x) \stackrel{?}{=}$

(iii) $\lim_{x\to 2} f(x) \stackrel{?}{=}$

(iv) $\lim_{x\to 0} f(x) \stackrel{?}{=}$

(v) $\lim_{x\to 3} f(x) \stackrel{?}{=}$

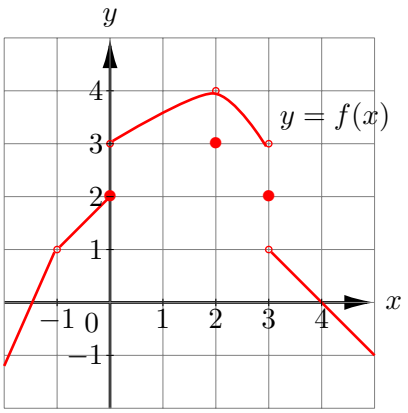

Figure 2

Exercise 2 Find $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$. Complete the following table of values to guess the limit and then use algebra to justify it (as in **A1**).

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$\dfrac{x^2-4}{x-2}$				**?**			

Q3: What are the basic **Limit Laws**?

A3:

Exercise 3 Determine the following limits using the properties of limits (i.e., limit laws) and by simplifying the expression, if necessary.

(i) $\lim_{x\to 5} x^4 \stackrel{?}{=}$

(ii) $\lim_{x\to 2} (5x^3 + 4x^2) \stackrel{?}{=}$

(iii) $\lim_{x\to 2} (5x^3 + 4x^2)\cdot(x^2 - 9) \stackrel{?}{=}$

(iv) $\lim_{x\to 2} \dfrac{x^2-9}{x-3} \stackrel{?}{=}$

(v) $\lim_{h\to 0} \dfrac{(h-2)^2 - 4}{h} \stackrel{?}{=}$

Exercise 4 If $f(x)$ is the function of Exercise 1 and $g(x) = 3x + 2$, then find the following limits:

(i) $\lim_{x\to 2} [f(x)\cdot g(x)] \stackrel{?}{=}$ Ans. 32

(ii) $\lim_{x\to 2} \sqrt{f(x)} \stackrel{?}{=}$ Ans. 2