Math 10250 Exam 1 Solutions – Spring 2008

1. $\lim_{t\to 5^-} P(t) = \lim_{t\to 5^-} \dfrac{20}{5-t} = +\infty$. Therefore the population **will** exceed 20 million.

2. $f(x)$ is defined provided $x - 5 \geq 0$ and $x \neq -1$. So $x \geq 5$ (this already excludes -1).

3. Slope of the graph of S is $0.5 = \dfrac{1}{2}$. So demand S increases as price p increases. Moreover, a \$2 **increase** in p will **increase** S by 1 thousand units.

4. $\lim_{x\to 2^+} \dfrac{x^2-4}{\sqrt{x-2}} = \lim_{x\to 2^+} \dfrac{(x-2)(x+2)}{\sqrt{x-2}} = \lim_{x\to 2^+} \sqrt{x-2}\cdot(x+2) = 0$

5. $g(-x) = \dfrac{(-x)^2+1}{(-x)^4+(-x)^2+1} = \dfrac{x^2+1}{x^4+x^2+1} = g(x)$ so $g(x)$ is an even function. Therefore the graph of $g(x)$ is symmetric about the y-axis. The graph of $g(x)$ is **NOT** symmetric about the origin.

6. The graph of $g(x)$ is obtained from the graph of $f(x)$ by translating the graph of $f(x)$ two units to the left horizontally, and three units down vertically. Therefore $g(x) = f(x+2) - 3$

7. Observe that $f(-2) < 0 < f(-1)$, and $f(2) < 0 < f(3)$. By the Intermediate value theorem, there must be a zero between $[-2,-1]$, and $[2,3]$ only.

8. Slope $= \dfrac{a^2-1}{1-a} = \dfrac{(a-1)(a+1)}{-(a-1)} = -(a+1)$.

9. Principal $P = 6000$, interest rate $r = 4\% = 0.04$, number of compounding $n = 12$, and maturity time $t = 21$ years. Balance $= 6000\left(1+\dfrac{0.04}{12}\right)^{(12)(21)} = 6000\left(1+\dfrac{0.04}{12}\right)^{252}$.

10. $\lim_{x\to 4^-} f(x) = 0$ and $\lim_{x\to 4^+} f(x) = 3$. Therefore $\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$. So $\lim_{x\to 4} f(x)$ does **NOT** exist.

11. (i) $P(x) = -2x^2+12x-8 = -2(x^2-6x)-8 = -2(x^2-6x+3^2-3^2)-8 = -2((t-3)^2-9)-8$
$= -2(t-3)^2+18-8 = -2(t-3)^2+10$
(ii) Maximum profit = 10 thousand dollars when $x = 3$ dozens.
(iii) $P(0) = R(0) - C(0) = -C(0) = -8 \Rightarrow C(0) = 8$ thousand dollars.

12. (A) $f(x) = \dfrac{x^2-9}{x^2-x-6} = \dfrac{(x-3)(x+3)}{(x-3)(x+2)} = \dfrac{x+3}{x+2}$ so there is one vertical asymptote $x = -2$. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{x^2-9}{x^2-x-6} = 1$ and $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \dfrac{x^2-9}{x^2-x-6} = 1$. Therefore the is one horizontal asymptote $y = 1$.

(B) For $f(x)$ to be continuous at $x = -1$, $f(-1) = \lim_{x\to-1} \dfrac{x^2-5x-6}{x^2-1} = \lim_{x\to-1} \dfrac{(x-6)(x+1)}{(x-1)(x+1)} = \lim_{x\to-1} \dfrac{x-6}{x-1} = \dfrac{-7}{-2} = \dfrac{7}{2}$. Therefore $c = 7/2 = 3.5$

13. (A) (i) Revenue = price · quantity. $R(x) = p\cdot x = \dfrac{12x}{x+2}$.

(ii) Cost $= C(x) = 2x+3$ (in thousands of dollars).
(iii) Profit = Revenue − Cost. $P(x) = R(x) - C(x) = \dfrac{12x}{x+2} - (2x+3) = \dfrac{12x}{x+2} - 2x - 3$.

(B) Let P be the principal. Then $\$10000 = P\left(1+\dfrac{0.04}{4}\right)^{4(10)} = P(1.01)^{40} \Rightarrow P = 10000(1.01)^{-40} = \$6716.53$.

14. (A) At equilibrium, $D(q) = S(q) \Rightarrow (q-5)^2+1 = q+8 \Rightarrow q^2-10q+25+1 = q+8 \Rightarrow q^2-10q+26-q-8 = 0 \Rightarrow q^2-11q+18 = 0 \Rightarrow (q-2)(q-9) = 0 \Rightarrow q = 2$ and $q = 9$ (Rejected because $0 < q < 5$). Therefore $q_e = 2$ and $p_e = S(q_e) = 2+8 = 10$.
(B) Set $P(t) = a\cdot b^t$. Given $P(0) = 1000$. Then $P(0) = a\cdot b^0 = a = 1000$. Also $P(2) = 2000$. Then $1000\cdot b^2 = 3000 \Rightarrow b^2 = 3 \Rightarrow b = \sqrt{3}$. Therefore $P(t) = 1000(\sqrt{3})^t = 1000(3)^{t/2}$.