

Multiple Choice

1. (5 pts.) The population of a bacteria $P(t)$ (in millions) is given by

$$P(t) = \frac{20}{5-t}$$

where $0 \leq t < 5$ is time in hours. Which of the statements below is FALSE?

- (a) The $P(t)$ is always increasing for $0 \leq t < 5$.
- (b) The $P(t)$ has a vertical asymptote.
- (c) The population increases in size **unboundedly** as time t approaches 5 hours.
- (d) The initial population is 4 million.
- (e) The population cannot exceed 20 million.

2. (5 pts.) Find the natural domain of the function $f(x) = \frac{\sqrt{x-5}}{x+1}$.

- (a) $x \leq 5$
- (b) $x \geq 5$
- (c) $x \neq -1$
- (d) $x > 5$
- (e) $x \neq -1, 5$

3. (5 pts.) The supply S (in thousand of units) of a brand of chocolate varies with its price p (in dollars) according to the function:

$$S(p) = 0.5p + 4$$

If the price of the chocolate is **increased** by \$2, what would happen to its supply?

- (a) Its supply would decrease by 1 thousand units.
- (b) Its supply would decrease by 5 hundred units.
- (c) Its supply would increase by 1 thousand units.
- (d) Its supply would increase by 2 thousand units.
- (e) There is not enough information to determine how its supply would change.

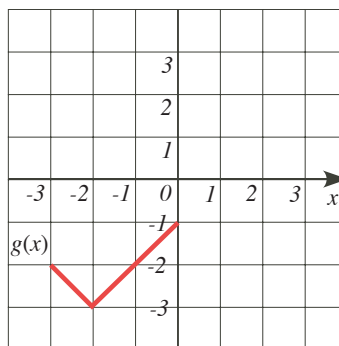
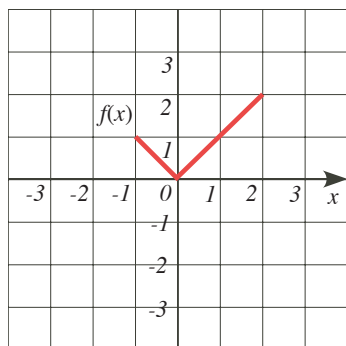
4. (5 pts.) Compute the following limit: $\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{\sqrt{x} - 2}$

- (a) 0
- (b) 4
- (c) Does not exist.
- (d) 1
- (e) -4

5. (5 pts.) Let $g(x) = \frac{x^2 + 1}{x^4 + x^2 + 3}$. Which of the following statements is **FALSE**?

- (a) The function $g(x)$ is symmetric about the y -axis.
- (b) The function $g(x)$ is even.
- (c) The function $g(x)$ is symmetric about the origin.
- (d) The natural domain of $g(x)$ is the set of all real values.
- (e) The value of $g(x)$ approaches zero as x gets unboundedly large.

6. (5 pts.) The graphs of $f(x)$ and $g(x)$ are given below.



Which of the following expression describes the relationship between $f(x)$ and $g(x)$?

- (a) $g(x) = f(x - 2) - 3$
- (b) $g(x) = f(x - 3) + 2$
- (c) $g(x) = f(x + 2) + 3$
- (d) $g(x) = f(x + 2) - 3$
- (e) $g(x) = f(x + 3) - 2$

7. (5 pts.) Assume that $f(x)$ is a continuous function on the interval $[-2, 3]$ with the following table of values

x	-2	-1	0	1	2	3
$f(x)$	1	-1	-2	-3	-1	2

In which of the following intervals

I. $[-2, -1]$

II. $[-1, 0]$

III. $[0, 1]$

IV. $[1, 2]$

V. $[2, 3]$

can you be sure that the function $f(x)$ has a root?

- (a) **II** and **IV** only.
- (b) **V** only.
- (c) **I** and **II** only.
- (d) **I** and **V** only.
- (e) **II** and **III** only.

8. (5 pts.) For any $a \neq -1, 1$, find the slope of the line joining $(a, 1)$ and $(1, a^2)$.

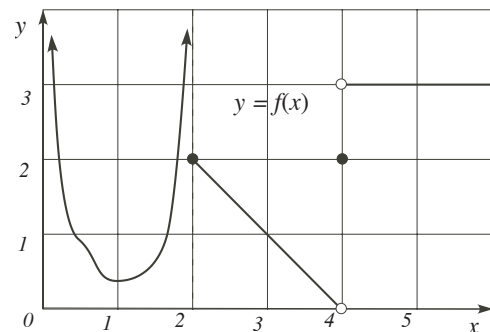
- (a) $a - 1$
- (b) $\frac{1}{a - 1}$
- (c) $-(a + 1)$
- (d) $a + 1$
- (e) $\frac{1}{-(a + 1)}$

9. (5 pts.) Suppose that on the day when his granddaughter is born, a man invested \$6000 in her name in an account paying 4% interest compounded **monthly**. How much will there be on her 21st birthday?

- (a) $6000 \left(1 + \frac{0.04}{12}\right)^{252}$ (b) $6000 \left(1 + \frac{0.04}{12}\right)^{12}$ (c) $6000 \left(1 + \frac{0.04}{12}\right)^{21}$
 (d) None of the above. (e) $6000 (1 + 0.04)^{21}$

10. (5 pts.) Let $f(x)$ be the function whose graph is shown below. Which of the following statements is **FALSE**?

- (a) $f(2) = 2$.
 (b) $\lim_{x \rightarrow 2^-} f(x) = +\infty$.
 (c) $\lim_{x \rightarrow 4^+} f(x)$ exists.
 (d) $f(x)$ is continuous at $x = 3$.
 (e) $f(x)$ is not continuous at $x = 2$.



Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) The profit, in thousands of dollars, from the sales of a certain ipod accessory is given by the formula

$$P(x) = -2x^2 + 12x - 8$$

where x is the number of dozens of accessory sold.

(i) (6 pts.) By completing the square, write $P(x)$ in the form $P(x) = a(x - h)^2 + k$. Show clearly all your steps.

(ii) (4 pts.) Write the maximum profit and the value of x when profit is maximum?

Maximum profit $\stackrel{?}{=} \underline{\hspace{2cm}}$ when $x \stackrel{?}{=} \underline{\hspace{2cm}}$.

(iii) (2 pts.) What is the **fixed** cost of the producing the accessory?

12. (12 pts.)

Part A. Find the **equations** of **all** vertical and horizontal asymptotes of the following functions. If there is none, circle "NONE".

	Equations	
(i) $f(x) = \frac{x^2 - 9}{x^2 - x - 6}$	Vertical: _____	NONE
	Horizontal: _____	NONE

Part B. (Independent from Part A.) Find the value of c such that the function $f(x)$ below is continuous at $x = -1$:

$$f(x) = \begin{cases} \frac{x^2 - 5x - 6}{x^2 - 1}, & \text{if } x \neq -1, 1 \\ c, & \text{if } x = -1 \end{cases}$$

Explain your work clearly using the limit definition of continuity.

Answer: _____

13. (12 pts.) (A) The price function for a model of jeans is

$$p = \frac{12}{x + 2}$$

in thousands of dollars and x is the number of jeans sold in units of hundreds. Suppose manufacturing has a fixed cost of three thousands, and the cost of manufacturing one unit of the jeans costs two thousand dollars, write down the following functions

(i) The revenue function $R(x)$.

(ii) The cost function $C(x)$.

(iii) The profit function $P(x)$.

(B) (Independent of A.) How much money should be put in an account paying 4% interest, compounded quarterly (four times a year), in order to have \$10,000 ten years from now?

14. (12 pts.)

(A) **Without using a calculator**, find the equilibrium point for the demand function

$$D(q) = (q - 5)^2 + 1 \quad \text{and} \quad S(q) = q + 8 \quad \text{for } 0 < q < 5$$

Find the equilibrium price p_e and equilibrium quantity q_e .

(B) (Independent of A.) A colony of fruit flies is growing exponentially. In 2 hours, the population grew from 1000 to 3000. Write a formula for the size of the population $P(t)$ of the colony of fruit flies as a function of time t in hours. (Hint: Write $P(t) = a \cdot b^t$)