Name ______________________________ Date ____________________

Math 10250 Review for Exam 2

1. If \$3,000 is deposited in an account paying 6% annual interest, compounded **continuously**. How long it will take for the balance to reach \$12,000? Ans. $t = \frac{\ln 2}{0.03}$

2. Polonium-210 has a decay constant of 0.004951, with time measured in days. How long does it take a given quantity of polonium-210 to decay to $\frac{1}{4}$ of the initial amount? Ans. $t \approx 280$

3. Use the definition of the derivative to find the derivative of each of the following functions.

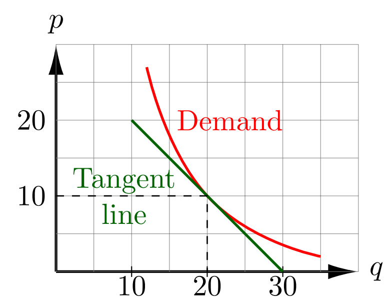
   (a) $f(x) = -3x^2 + 4$ (b) $f(x) = \dfrac{1}{3x+8}$ (c) $f(x) = 4\sqrt{x} + 5$

4. Find: (a) $\displaystyle\lim_{h\to 0} \frac{(7+h)^{10} - 7^{10}}{h}$ and (b) $\displaystyle\lim_{h\to 0} \frac{10^{7+h} - 10^7}{h}$ (Hint: Think derivative!) (Ans: (a) $10(7)^9$; (b) $10^7 \ln 10$)

5. The demand curve of a certain product is shown in next figure. The price p is measured in dollars and the quantity q in millions of units.

   (a) Find the marginal revenue MR at $q = 20$. (Ans: -10)

   (b) Use linear approximation to estimate $R(20.1)$. (Ans: 199)



6. Assume that a population grows according to the (exponential) model $\frac{dP}{dt} = 0.02P$. If the population now is 5 millions, use linear approximation to estimate this population 10 years later. (Ans: 6 millions)

7. Assume that a population grows according to the model $\frac{dP}{dt} = 0.02P(1 - 0.1P)$. If the population now is 5 millions, use linear approximation to estimate this population 10 years later. (Ans: 5.5 millions)

8. A ball is thrown into the air and its height in feet (measured from the ground) after t seconds is given by $s = -16t^2 + 32t + 48$ until it hits the ground.

   (a) What is the initial height of the ball? (Ans: 48 ft)

   (b) What is its velocity at the end of 1, and 1.5 seconds? In what direction (up or down) is it moving at the end of 1 and 1.5 second? (Ans: $v(t) = -32t + 32$)

   (c) At what time does the ball hit the ground? (Ans: 3 sec.)

   (d) What is the ball's acceleration at the end of 0.5 seconds? What is the ball's acceleration after 1 second? (Ans: -32 ft/s^2)

9. $\left(x^4 - e^{3x}\right)''' \stackrel{?}{=}$ (Ans: $24x - 27e^{3x}$)

10. The demand for an item is $p = 80 - 0.2x$ and its cost function is $C(x) = 20x + 100$, where x is the quantity of the item. Find the marginal revenue, cost and profit. If every item made is sold, should the company increase production to increase profit when $x = 100$? when $x = 200$? Explain. (Ans: $R'(x) = 80 - 0.4x$, $C'(x) = 20$, $P'(x) = R'(x) - C'(x)$)

11. Let $f(x) = x^3 g(2/x)$. If $g(1) = 3$ and $g'(1) = 10$, then find $f'(2)$. (Ans: -4)

12. The GDP of a country at the beginning of 2006 was \$500 billion dollars and it was growing at a rate of \$20 billion per year. Use tangent line approximation to estimate the GDP of this country at the end of the third quarter. (Hint: Let $G(t)$ be the GDP. Then $G(0) = 500$ and $G'(0) = 20$. Thus $G(3/4) \approx 515$)

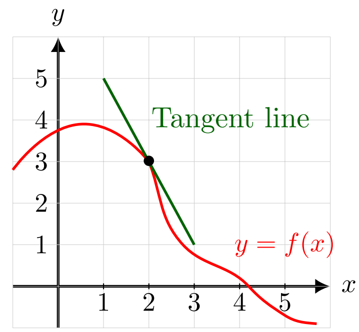
13. Given the graph of $f(x)$, find each of the following derivatives below.

(a) If $p(x) = f(x) \cdot x^3$ then $p'(2) \stackrel{?}{=}$

(b) If $q(x) = \dfrac{f(x)}{f(x)+1}$ then $q'(2) \stackrel{?}{=}$

(c) If $r(x) = \ln(f(x)+e)$ then $r'(2) \stackrel{?}{=}$

(d) $s(x) = e^{f(x)} + (f(x))^3$ then $s'(2) \stackrel{?}{=}$



Ans: (a) 20, (b) $-1/8$, (c) $\frac{-2}{3+e}$, (d) $-2e^3 - 54$

14. Use the approximation $\log_2 3 \approx 1.585$ and $\log_2 5 \approx 2.322$ to approximate the following:

(a) $\log_2 30 \stackrel{?}{\approx}$ Ans.4.907

(b) $\log_2 15 \stackrel{?}{\approx}$ Ans.3.907

(c) $\log_2(9/10) \stackrel{?}{\approx}$ Ans.-0.152

15. A chain of gourmet food stores sells a delicacy prepared from a rare fish species. Suppose that the amount of delicacy available at any time during the 16-week season is given by

$$w = 1000te^{-0.02t^2}, \qquad 0 \le t \le 16,$$

where w is the number of pounds and t is the time in weeks. Suppose the price per pound is $p = 500 - 0.08w$. How fast (in dollars per week) is the **revenue** from this delicacy changing at the end of 8 weeks? (Ans: $-62,506.99$ dollars/week)

16. Suppose a rectangular tank, whose base is a square of length 5 feet, is filling with water at the rate of 0.5 cubic feet per minute. How fast is the water level rising? (Ans: 1/50 ft/min)

17. You have just brought your Starbucks coffee into your room, which is kept at the temperature of 70°F. Five minutes later the temperature of the coffee is 190°F and is decreasing at a rate of 3°F per minute. Write a differential equation modeling the temperature $H(t)$ of your coffee. (a) Find $H(5)$ and $H'(5)$. (b) Is $H'(5)$ positive or negative? What does this say? (c) Finally, find a formula for $H(t)$. (Ans: $H(t) = 70 + 120e^{-0.025(t-5)}$)

18. The radioactive carbon in a piece of wood taken from an ancient cave decays at the rate of 6 disintegrations per minute (dpm), while the radioactive carbon in a similar sample of fresh wood decays at the rate of 8 dpm. Using 5,568 years as the half-life of radioactive carbon, estimate the age of the wood. (Ans: Age$\approx$ 2310.93 years)

19. Compute: (a) $\frac{d^2}{dx^2}\, e^{x^2+1}$ (b) $\frac{d}{dt}\, \frac{\ln(t^4)}{t^2}$

20. For a differentiable function $f(x)$ with $f(8.1) = 23.8$ and $f(7.9) = 23.4$ estimate $f'(8)$. (Ans: $f'(8) \approx 2$)

21. Draw the graph of a function defined on $[0,5]$ which is **not** differentiable **only** at three points in $(0,5)$ and **not** continuous **only** at one point.

22. Use implicit differentiation to find $\dfrac{dy}{dx}$ when $y^4 - ye^{5x} = 14$. Ans: $\frac{dy}{dx} = \frac{5ye^{5x}}{4y^3 - e^{5x}}$

23. Find the equation of the tangent line to the curve $x^2 - xy^2 = e^y$ at the point $(1,0)$. (Ans: $y = 2(x-1)$)

24. Water is leaking out of a cylindrical tank, whose base is a circle of radius 0.4 feet, at the rate of 0.16 cubic feet per minute. Find dh/dt. (Ans: $-1/\pi$)

25. Give the definition of the derivative and list its different names. Then, apply the definition to compute the derivative of your favorite function.

26. Write an one paragraph summary about the things you learned in Math 10250 thus far.