Name ______________________________ Date ____________________

Math 10250 Review for Exam 1

1. (a) Determine the natural domain of $f(x) = \dfrac{2-x}{x-1}$, find also its inverse $g(x)$. Ans. $x \neq 1$; $g(x) = \frac{x+2}{x+1}$

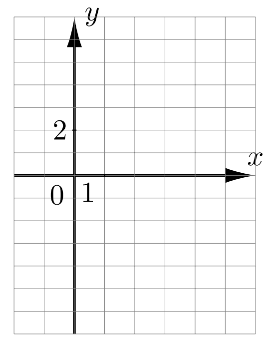
   (b) What is the natural domain of $f(x) = \sqrt{3-2x}$, and what is its inverse? Ans. $x \leq 3/2$; $g(x) = \frac{3}{2} - \frac{1}{2}x^2$

2. A brand of sunglasses selling for \$50 each has a demand of 1,500 units. However, when the price is **increased** by \$5, its demand is **decreased** by 100 units. Find its demand assuming that is a linear function. Ans. $q = D(p) = -20p + 2,500$

3. Complete the square for each quadratic and then sketch its graph.

   (i) $f(x) = -3x^2 + 12x$

   (ii) $f(x) = 2x^2 - 12x + 10$.



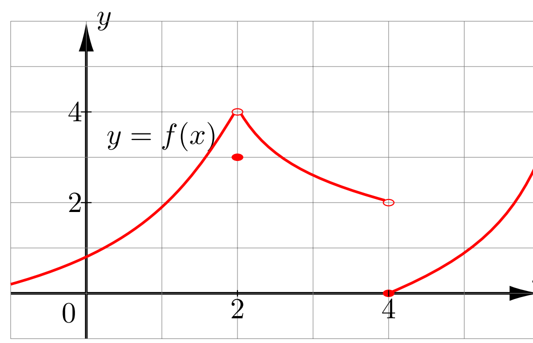
Ans. (i) $f(x) = -3(x-2)^2 + 12$; (ii) $f(x) = 2(x-3)^2 - 8$

4. When the price p of a particular computer is \$2,000 then the demand x is 50,000 units per week. However, when the price drops by \$500 then the demand rises by 25,000 units. On the cost side, the company making these computers has \$40,000,000 fixed cost and \$600 expenses per unit. Assuming that the demand is linear, find the profit function P in terms of x and its maximum value.

   Ans. $P = -0.02x^2 + 2{,}400x - 40{,}000{,}000 = -0.02(x-60,000)^2 + 32{,}000{,}000$

5. (a) $\lim\limits_{h\to 0} \dfrac{5(1+h)^2 - 5}{h} \stackrel{?}{=}$ (b) $\lim\limits_{h\to 0} \dfrac{\frac{1}{2+h} - \frac{1}{2}}{h} \stackrel{?}{=}$ Ans. (a) 10; (b) $-\frac{1}{4}$

6. The graph of the function $f(x)$ is given in the next Figure.



   Which of the following statements is **NOT** true?
   (a) $\lim\limits_{x\to 2} f(x) = 4$
   (b) $\lim\limits_{x\to 4^-} f(x) = 2$ and $\lim\limits_{x\to 4^+} f(x) = 0$
   (c) $f(x)$ has limit at $x = 4$.
   (d) $f(x)$ is continuous except at the points $x = 2, 4$.
   (e) $\lim\limits_{x\to 0} f(x) = 1$.

   Ans. c

7. Let $f(x)$ be the function whose graph is shown above. Compute $\lim\limits_{x\to 2} \dfrac{5x^2 + 3x + 4}{\sqrt{x^3 f(x) + 68}}$. Ans. 3

8. If $x \neq 2$ then $f(x) = \dfrac{x^2 + 2x - 8}{x - 2}$. Define $f(2)$ so that $f(x)$ is a continuous function. Ans. $f(2) = 6$

9. In which of the following intervals you can be sure that the function $f(x) = x^4 + 2x^3 - 3x^2 - 2x + 3$ takes the value 2? (i.e the equation $f(x) = 2$ has a solution.) $[-3,-2], [-2,-1], [-1,0], [0,1], [1,2], [2,3]$

   Ans. $[-3,-2]$, $[-1.0]$, $[0,1]$, $[1,2]$

10. For each function below, find vertical asymptote(s), horizontal asymptote(s), y-intercept, its zero(s), and then sketch its graph. Ans. (a) v.a: $x = \pm 1$; h.a: $y = 1$; zeros: $x = \pm 4$, y-intercept: 16; (b) v.a: $x = -1$; h.a: $y = 0$; zeroes: None, y-intercept: $y = 1$

    (a) $f(x) = \dfrac{x^2 - 16}{x^2 - 1}$ (b) $f(x) = \dfrac{x - 4}{x^2 - 3x - 4}$

11. Suppose that you put \$100 in an account paying 2% annual interest, compounded daily. How much will you have at the end of 1 day? 2 days? and 3 days? Ans. $100\left(1 + \frac{0.02}{365}\right)$; $100\left(1 + \frac{0.02}{365}\right)^2$; $100\left(1 + \frac{0.02}{365}\right)^3$

12. Suppose that you have an account paying interest, compounded weekly, has balance given by $P(t) = 8000(1.0004)^{52t}$. What is its principal and annual interest rate? Ans. $P = 8000$; $r = 2.08\%$

13. Imagine that you just got that great jobs, which among many good things it offers you one million dollars 45 years from now as a retirement benefit. What is the present value of this amount assuming annual interest of 6% compunded **daily**.

    Ans. \$67,220

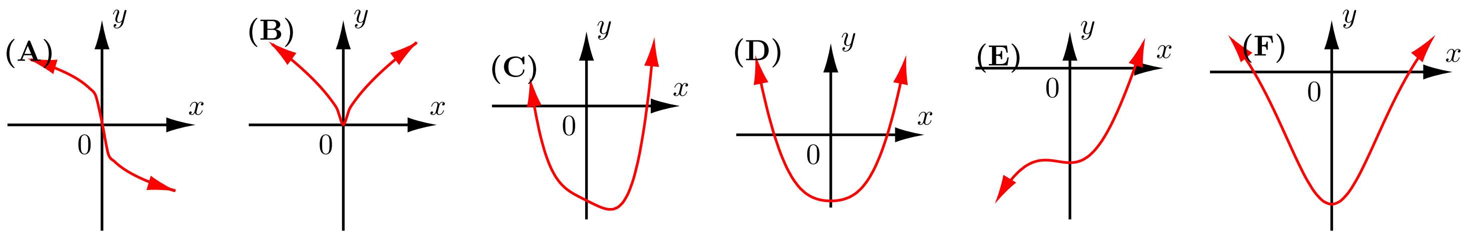
14. Find the Future value in 20 years of \$100 deposited into an account paying 5% interest, compounded **continuously**.
Ans. $FV = 100e$

15. A population of bacteria on a growing medium is initially 10 million. Three hours later the number of bacteria is numbered at 15 million. Write down a formula for the population $P(t)$ at time t in hours if the population is growing exponentially.
Ans. $P(t) = 10(1.145)^t$

16. Match the following functions with the given graphs without using your calculator:

$f_1(x) = -x^{1/3}$ $\quad f_2(x) = x^{2/3}$ $\quad f_3(x) = x^4 - x - 5$

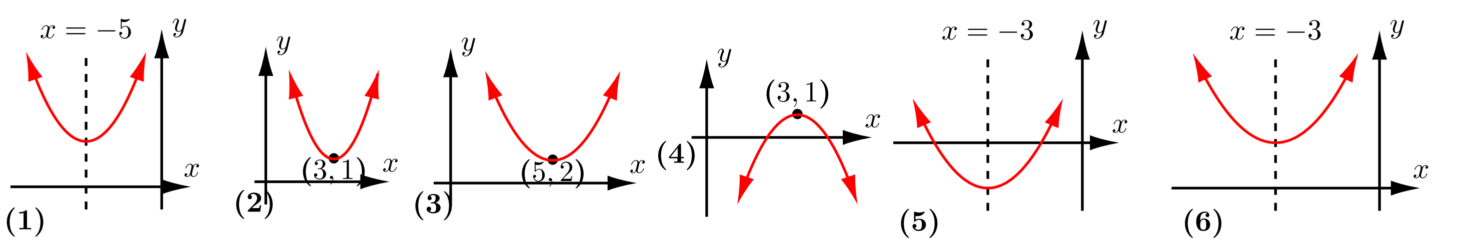
$f_4(x) = \dfrac{5x^4 - 25}{x^2 + 5}$ $\quad f_5(x) = \dfrac{5x^3 - 25}{x^2 + 5}$ $\quad f_6(x) = \dfrac{5x^2 - 25}{x^2 + 5}$



17. Match the graphs to the given quadratic functions. Some graphs are superfluous.

$f_1(x) = (x-5)^2 + 2$ $\quad f_2(x) = a(x-3)^2 + 1 \quad (a < 0)$

$f_3(x) = b(x+3)^2 - 1 \quad (b > 0)$ $\quad f_4(x) = (x+5)^2 + 2$



18. A private health club has determined that the number of members depends on the price of a membership, and they are related by an equation of the form $q = 3000 - 20p$, where q is the number of members and p is the annual price of a membership. The club has a fixed costs of \$20,000 per year plus an average annual cost of \$40 per member.
    (a) Write the club's revenue R as a function of the price p. Ans. $R = 3000p - 20p^2$
    (b) Write the club's profit P as a function of the price p. Ans. $P = -20p^2 + 3800p - 140000$
    (c) What membership price should the club set to maximize its profit? Ans. \$95
    (d) Find the break-even point. Interpret your answer. Ans. \$50 and \$140

19. Find the equilibrium price p_e and equilibrium quantity q_e for each pair of demand and supply functions. Make a sketch of the graphs marking the coordinates of intersection point.
    (a) $q = D(p) = -p + 12$ and $q = S(p) = 2p - 3$ for $p \geq 0$ Ans. $p_e = 5$, $q_e = 7$
    (b) $p = D(q) = 0.005(q - 100)^2$ and $p = S(q) = 0.1q + 2$ for $0 \leq q \leq 100$ Ans. $p_e = 8$, $q_e = 60$
    (c) $q = D(p) = \dfrac{8}{p+1}$ and $q = S(p) = \dfrac{1}{3}p + 1$ Ans. $p_e = 3$, $q_e = 2$

20. If \$3,000 is deposited in an account paying 6% annual interest, compounded **continuously**. How long it will take for the balance to reach \$6,000? Ans. $t = \frac{\ln 2}{0.06}$

21. How much money must you invest in an account paying 3% annual interest compounded **continuously** in order to have a balance of \$20,000 in 10 years? (Ans. $20000e^{-0.3}$)

22. Compare the magnitude 8.0 earthquake which occurred near Samoan on September 29, 2009, with the 7.0 earthquake which occurred in San Francisco on October 17, 1989. (Hint: See Example 2.3.2, p. 140)