Synthesis of Robust Controller for Spacecraft Pitch Control System

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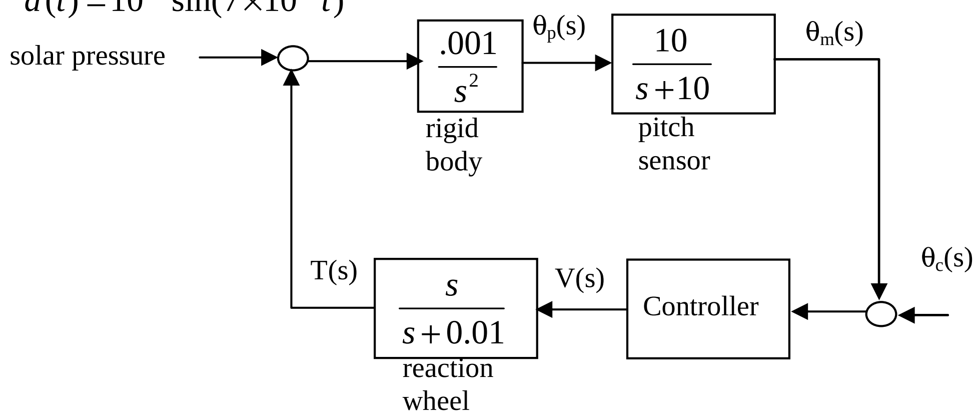
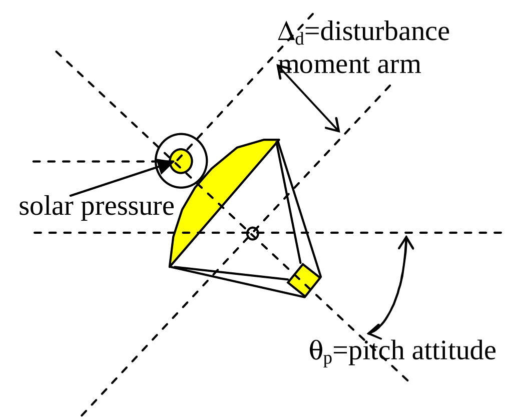
Abstract: This lecture uses loopshaping to design a robust controller for a "real-life" application involving spacecraft pitch attitude control.

Spacecraft Attitude Dynamics

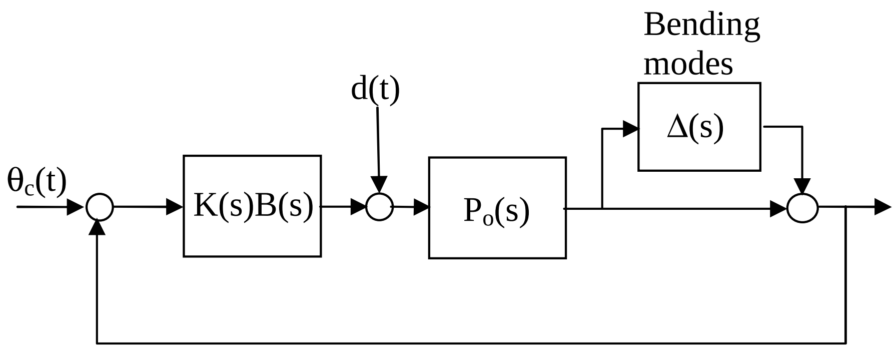
- reaction wheel
- solar disturbance (24 hour period)
- pitch attitude sensor (low pass filter)
- rigid body dynamics = double integrator

$$d(t) = 10^{-5} \sin(7 \times 10^{-5} t)$$





Bending Mode Dynamics

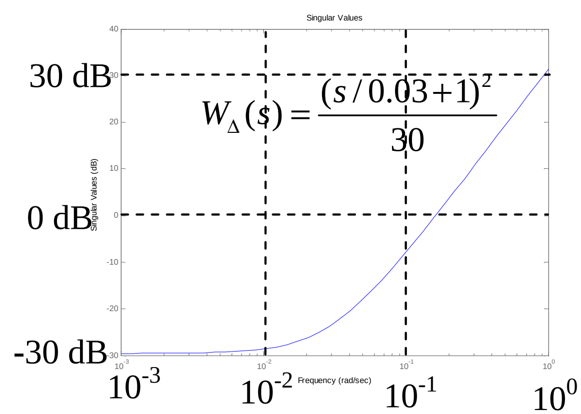


$$P_o(s) = \frac{10}{s+10}\frac{0.001}{s^2}$$

$$B(s) = \frac{10}{s+10}$$

$K(s) =$ Controller

$G_o(s) = P_o(s)B(s) =$ nominal plant



Performance Weighting Function

- Bandwidth of closed loop system as large as possible
- Oscillatory responses at a minimum
- low sensitivity to disturbances (solar pressure)
  0.01° pitch attitude error (0.17 mrad)

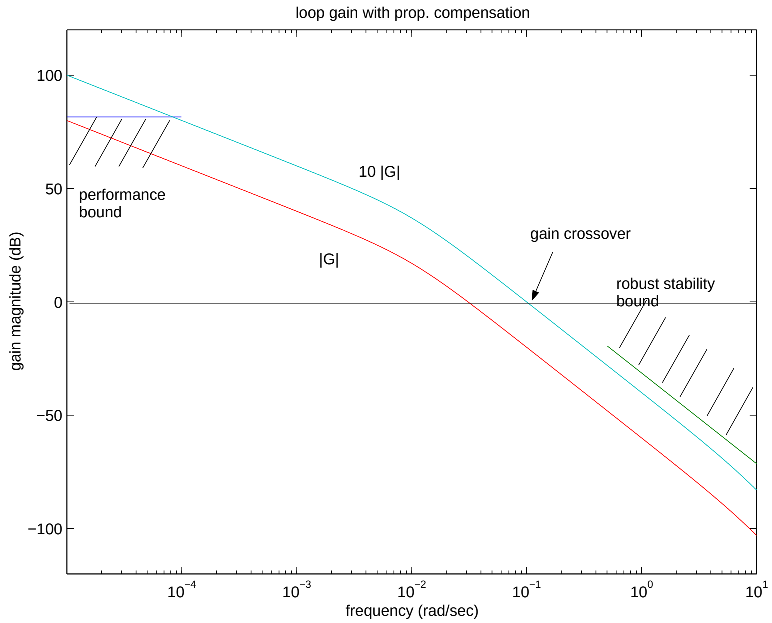
Disturbance requirement is enforced at Earth's orbital frequency $\omega_e = 7 \times 10^5$ rad/sec.

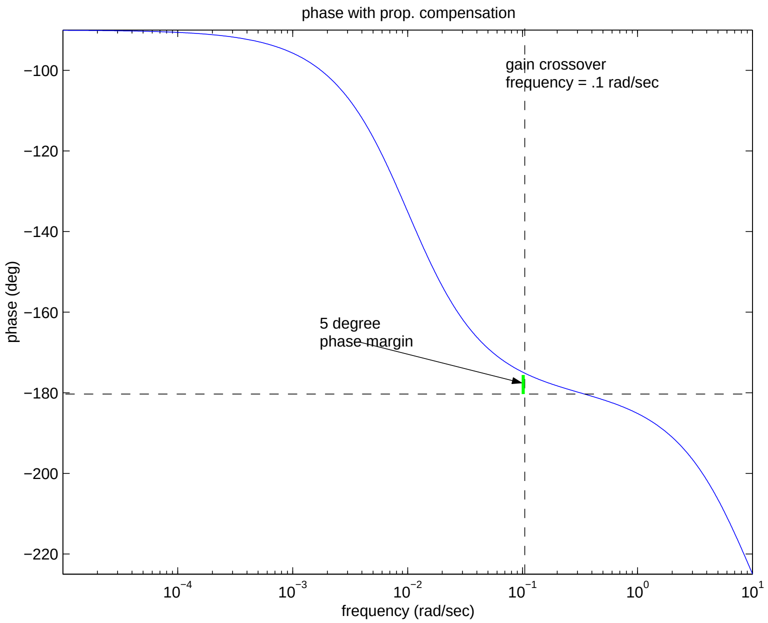
$$|\theta_m| = \frac{\frac{0.01}{\omega_e^2 \sqrt{\omega_e^2 + 100} 10^{-5}}}{|1 + K(j\omega_e)G(j\omega_e)|} < 1.7 \times 10^{-4} \Rightarrow |K(j\omega_e)G(j\omega_i)| \geq 12,000$$

So we require the closed loop gain KG to be less than 12,000 at ω_e. The associated weighting function is

$$|W_p(j\omega)| = \begin{cases} 12,000 & \omega < \omega_e \\ 0 & \text{otherwise} \end{cases}$$

Proportional Compensation Block

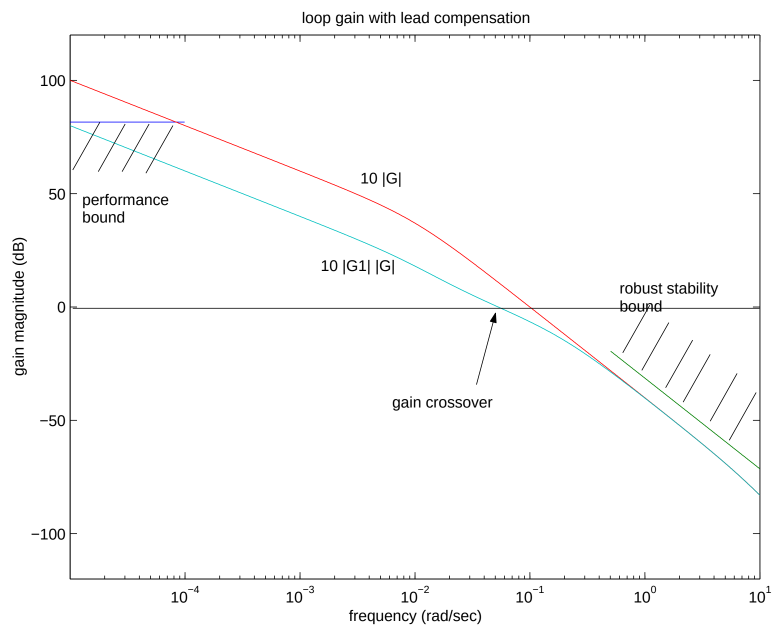


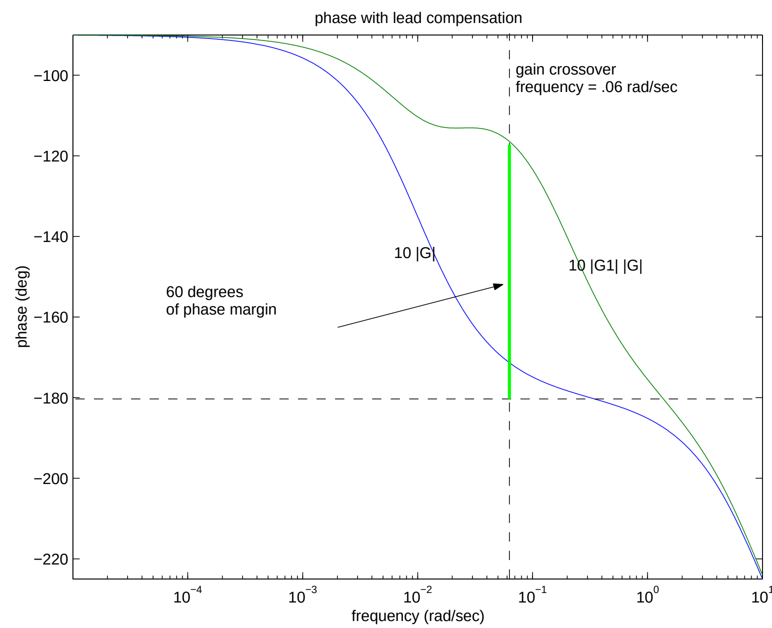


Initial Compensator Block is Proportional Gain ($K(s) = 10$).

Very small phase margin (5°) which implies large overshoot (20 dB)

Lead Compensation Block

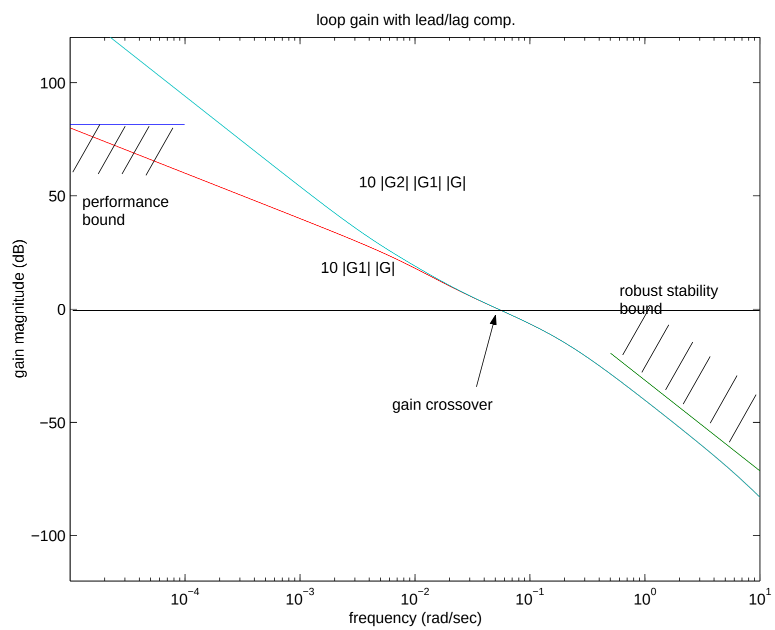


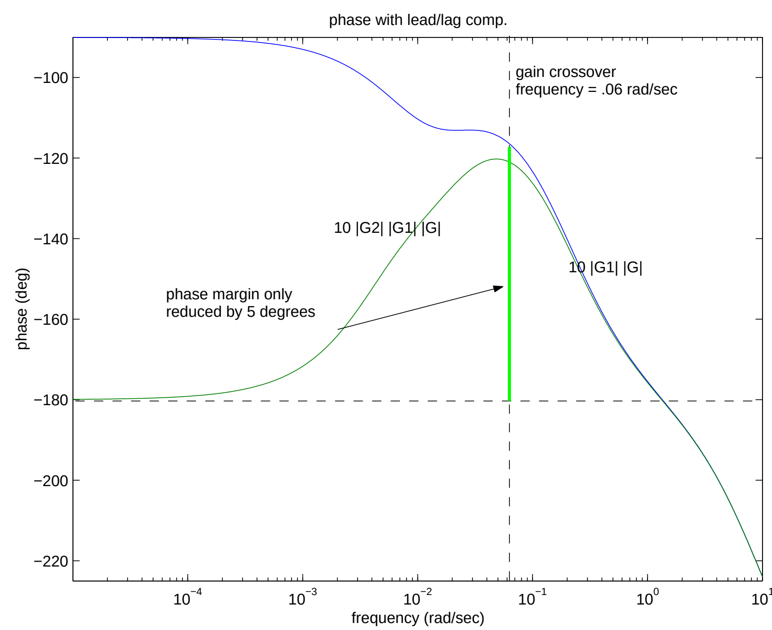


Add 55° of phase lead at $\omega_\ell = 0.06$ rad/sec.

$K_{\text{lead}}(s) = \frac{s+0.019}{s+0.19}$ achieves 60° phase margin but violates LF bound.

Lag Compensation Block

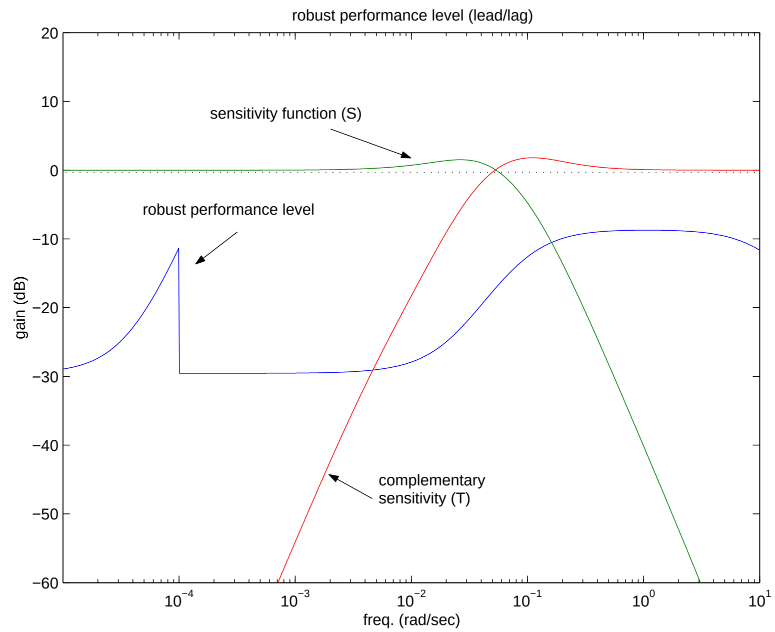


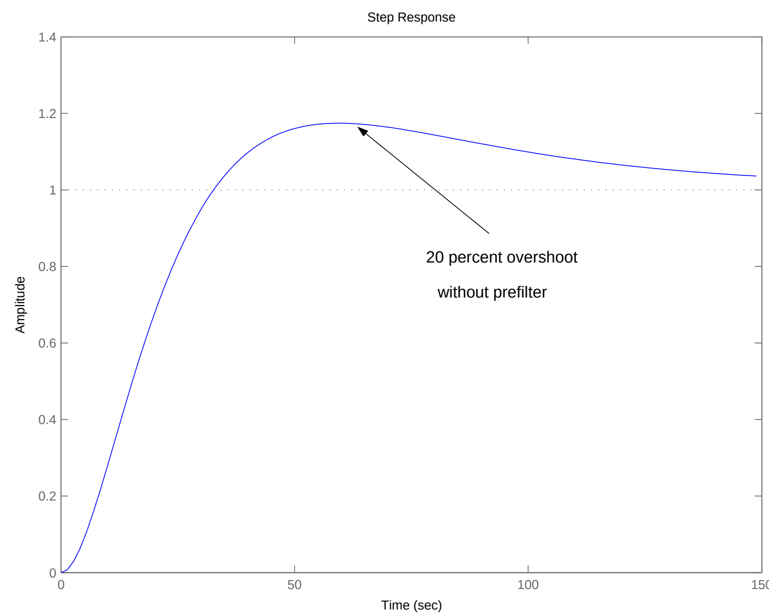


Use lag compensator $K_{\text{lag}}(s) = \frac{s+0.005}{s}$ to increase LF loop gain.

Note slight loss of phase margin.

Double Check Design

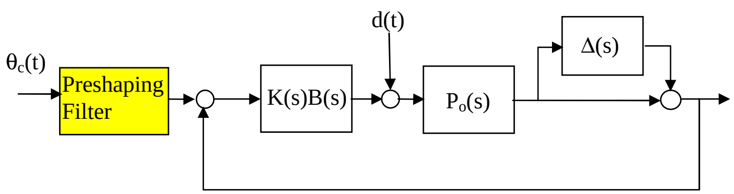




Plot $||W_p S_o| + |W_\Delta T_o||$ to see if it is less than one.

Plot step response to double check transient behavior.

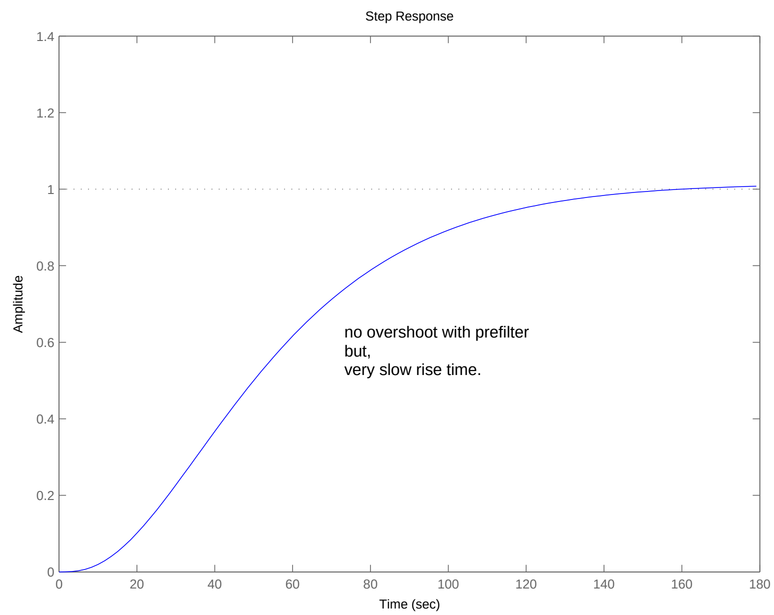
Preshaping Filter



$$P(s) = \frac{0.019}{s + 0.019}$$

Command shaping prefilters are synthesized by "trial and error"

Or by solving a "model-matching" problem.



Transient response is often determined by zeros, which cannot be moved through feedback.

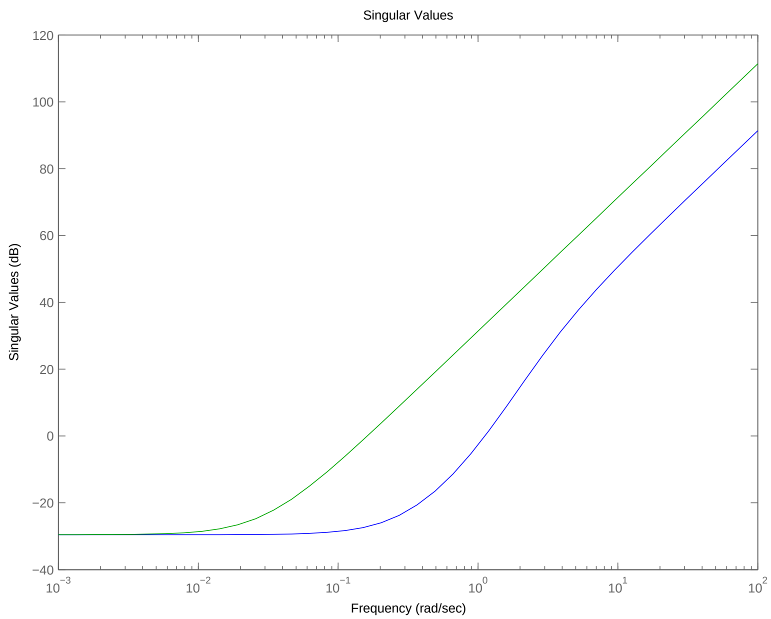
Two parameter feedback system uses a command preshaping filter to shape transient response. But response is very slow!!!

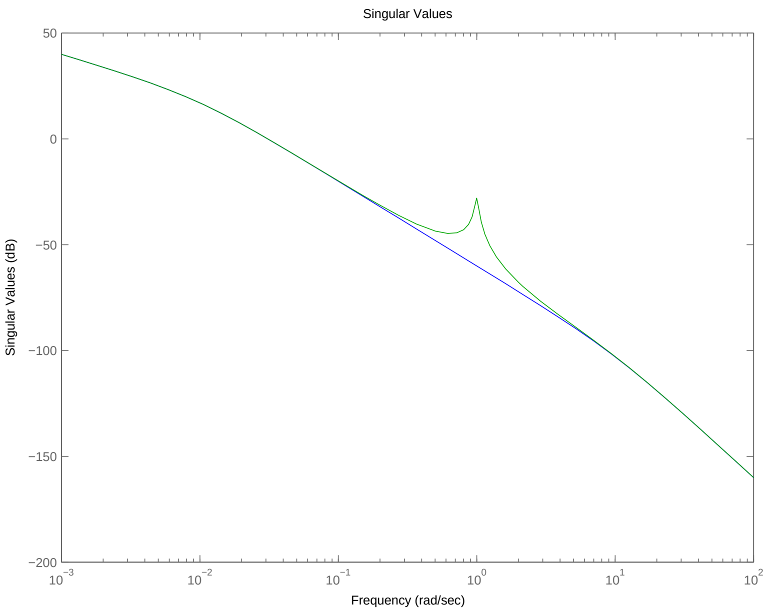
Design Process is Iterative!

- Previous design achieves the specifications, but settling time is 120 seconds. This translates to "long outage periods" after a command change in the satellite's attitude.
- Customer is not happy with this design and demands you do better. However, it is highly unlikely that the design can be improved with the current level of model uncertainty.
- The only way to improve this is to actually use a better model for the bending mode.
- So go back to structural engineers and do more testing to obtain a better model of the uncertainty,

$$
\begin{aligned}
G_o(s) &= \frac{10}{10^3 s(s+0.01)(s+10)} \frac{(s+1)^2}{s^2+0.04s+1} \\
W_\Delta(s) &= \frac{(1-(s/0.3).^2)(1-s^2)}{30(1+(s/10)^2)}
\end{aligned}
$$

New Plant Model

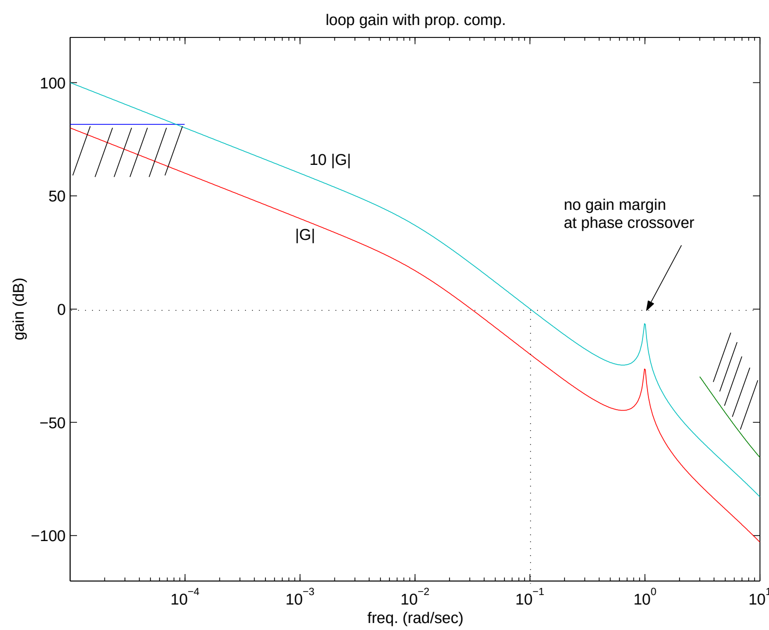


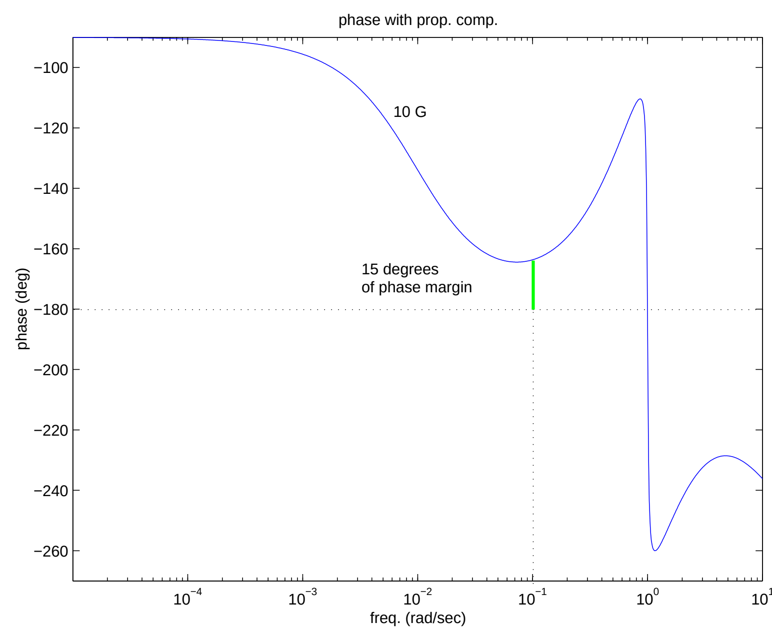


The revised nominal plant model has included a single resonant mode.

Revised W_Δ increases the size of the transition region.

Redesign with New Plant

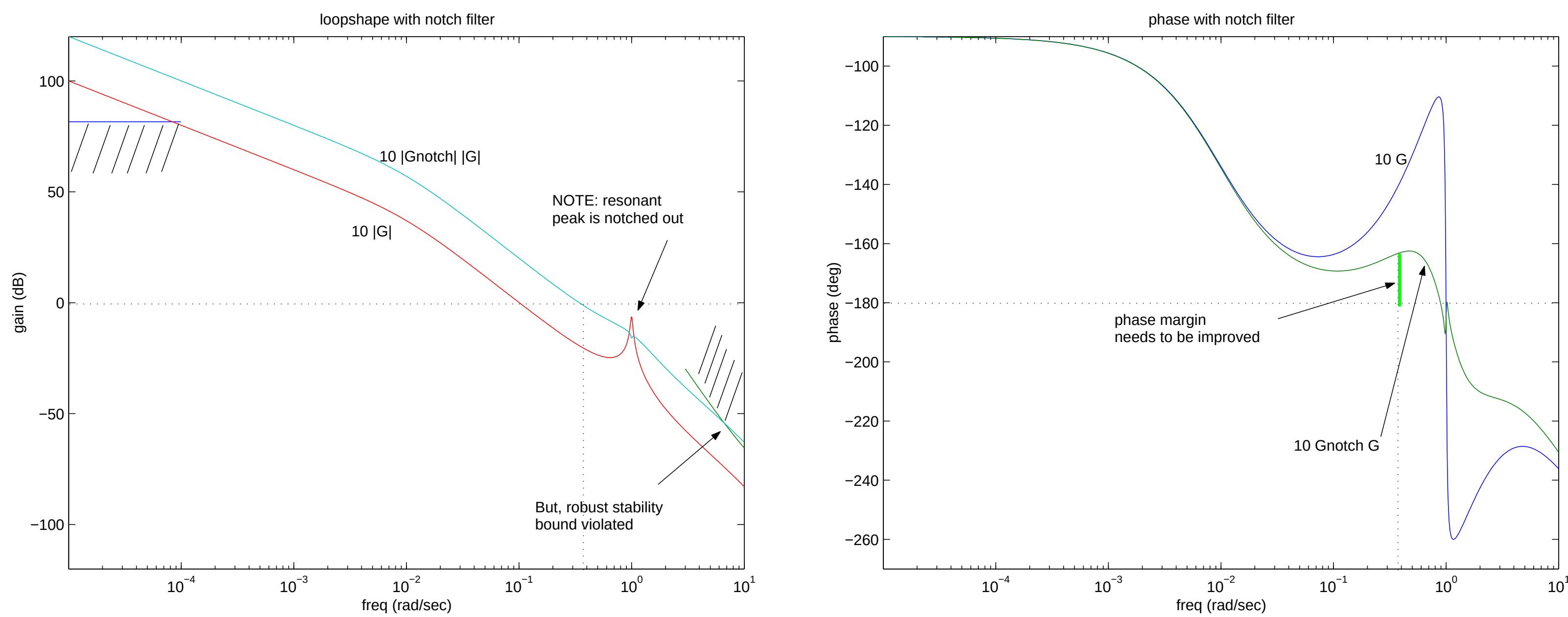




Again use propotional gain of 10.

Phase margin is now 15°, but gain margin is almost all gone!!!

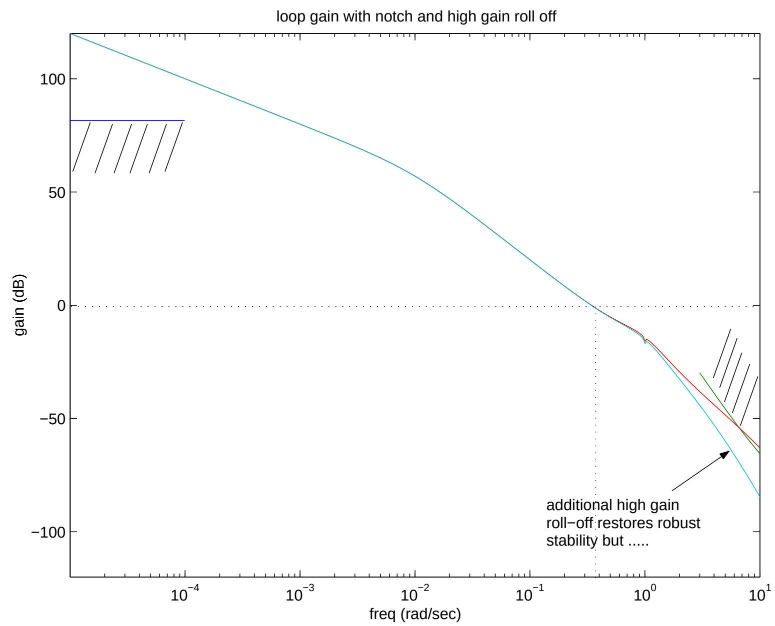
Notching out Bending Mode

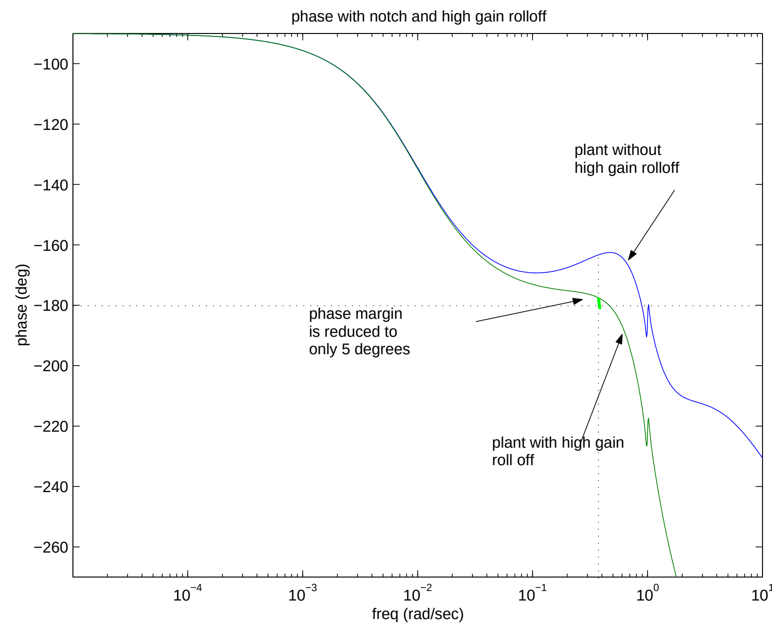


Place a notch at 1 rad/sec with a depth of -30 dB, $K_n(s) = \frac{s^2+0.0316s+1}{s^2+s+1}$.

High frequency robustness bound is violated and phse margin is too small.

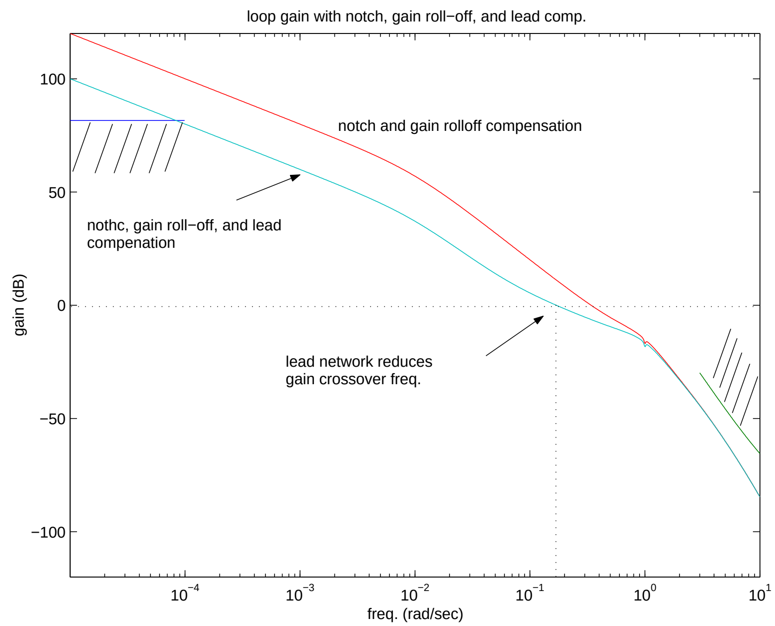
High gain Roll-off Filter

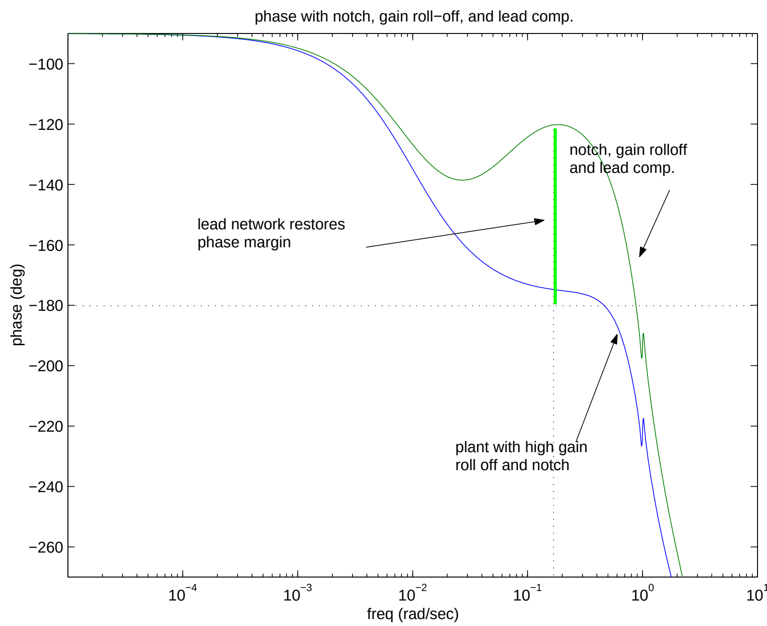




High gain roll off $K_r(s) = \frac{9}{(s+3)^2}$ adds additional 40 dB of roll-off ater 3 rad/sec. Thereby recovering robustness constraint. But takes away even more phase margin.

Lead Filter





Lead filter, $K_l(s) = \frac{s+0.063}{s+0.63}$, recovers phase margin.

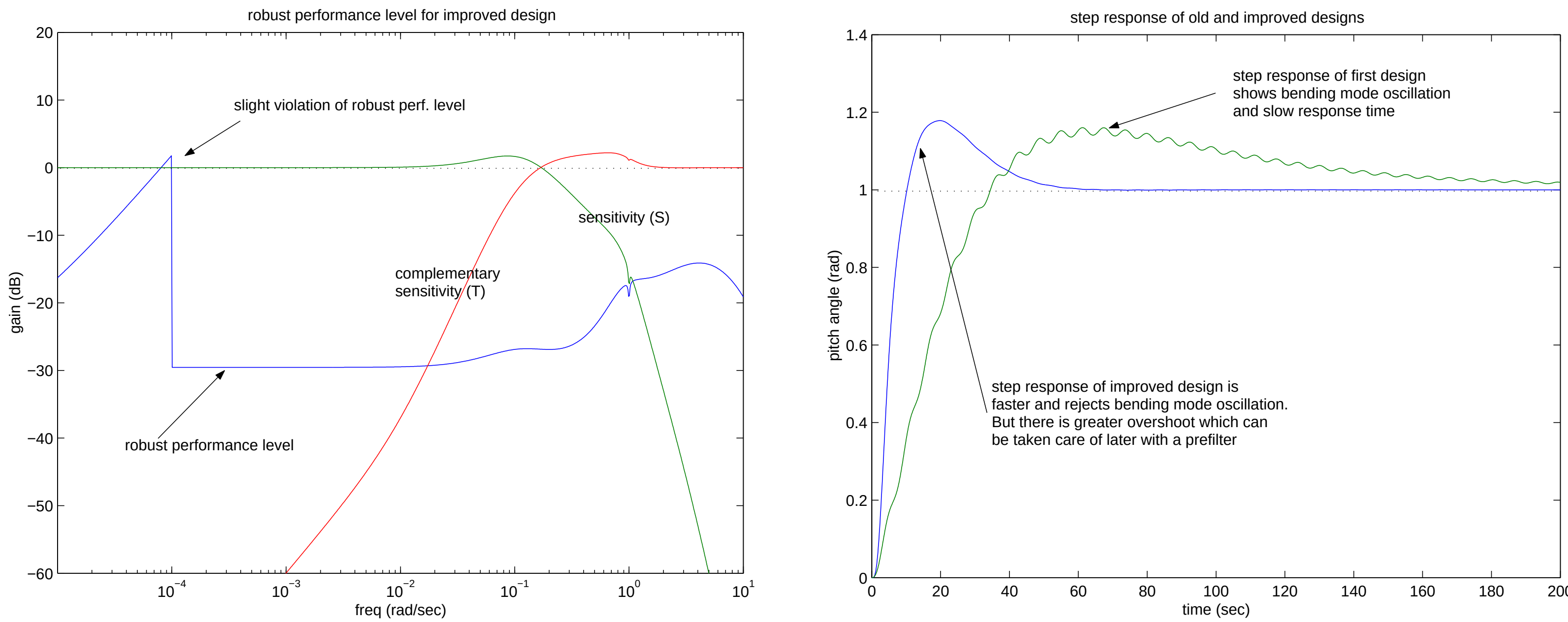
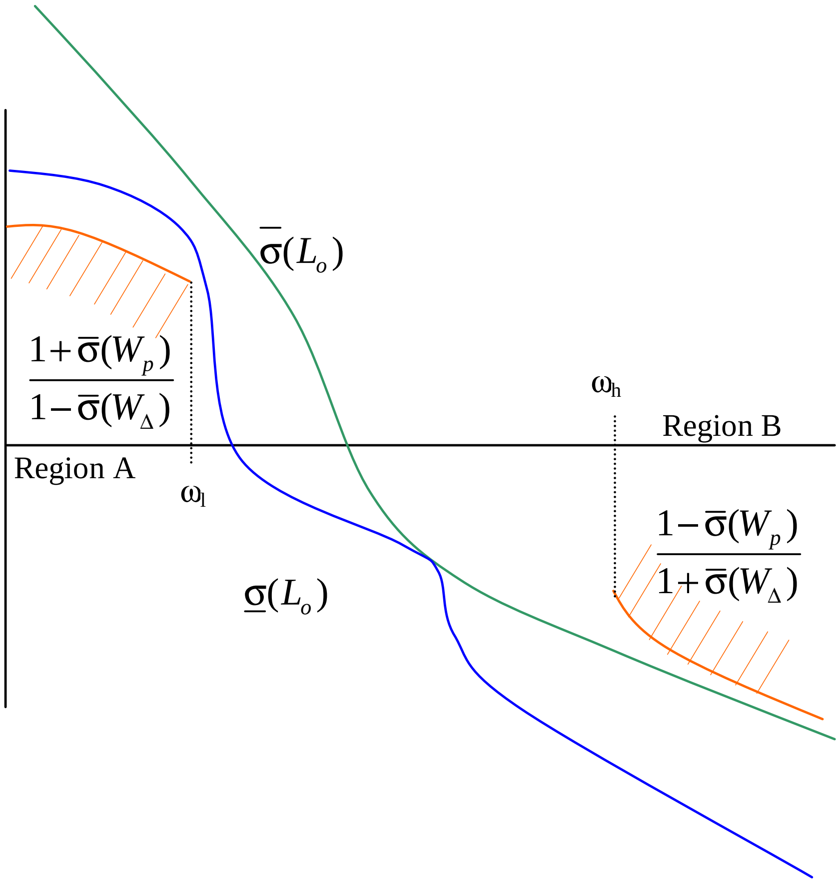


Figure 1: Performance bound and step responses

Slight violation of robust performance constraint, which we'll neglect.

Faster response (30 second settling time), but still have 20 percent overshoot that will have to be removed using a command shaping prefilter.

Multivariable Loopshaping



We can shape the singular values of a MIMO plant's loop using principles similar to those employed in scalar loopshaping.

But we will have problems in assuring the resulting loopshape has internal stability.

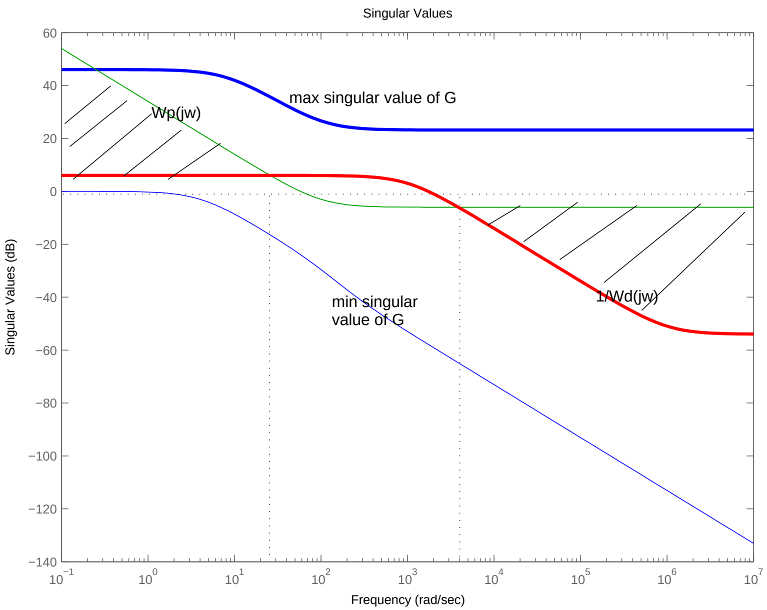
Example

MIMO loopshaping problem:

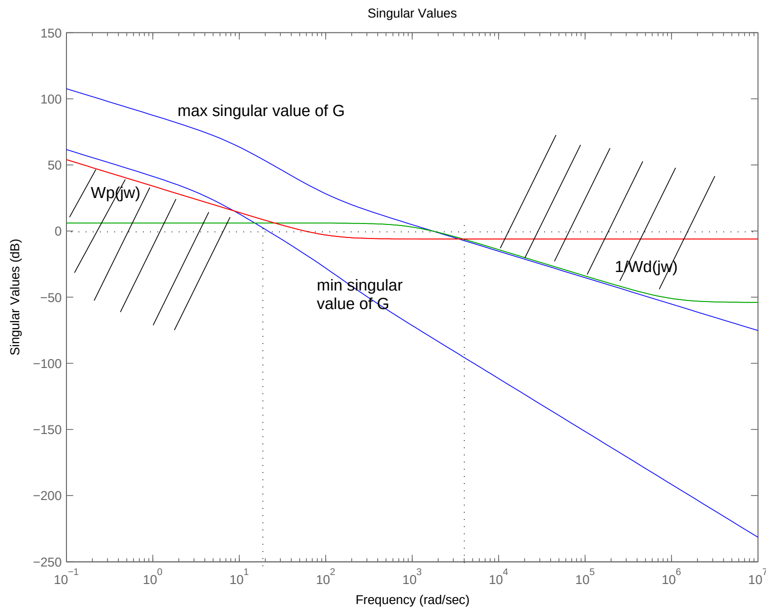
$$G_o(s) = \begin{bmatrix} \frac{4}{s+4} & 0 \\ \frac{4s(3s+10)}{(s+4)(s+8)} & \frac{8(s-200)}{s+8} \end{bmatrix}$$

$$W_\Delta(s) = \frac{10^{-3}s+1}{2(10^{-6}s+1)}\mathbf{I}$$

$$W_p(s) = \frac{s+100}{2s}\mathbf{I}$$



Use an Integrator



Integrator controller, $K(s) = \begin{bmatrix} \frac{120}{s} & 0 \\ 0 & \frac{120}{s} \end{bmatrix}$. This control design is unstable.

We need a more powerful tool. That tool is the $\mathcal{H}_\infty$ synthesis.