3/30/06

Multivariable Loopshaping and H_∞ Control

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Abstract: This lecture uses shows how to use H_∞ controller synthesis tools in Matlab to solve the robust performance problem.

Problem

$$\mathbf{G}_o(s) = \begin{bmatrix} \frac{4}{s+4} & 0 \\ \frac{4s(3s+16)}{(s+4)(s+8)} & \frac{8(s-200)}{s+8} \end{bmatrix}$$

$$\mathbf{W}_\Delta(s) = \frac{10^{-3}s+1}{2(10^{-6}s+1)}\mathbf{I}$$

$$\mathbf{W}_p(s) = \frac{s+100}{2s}\mathbf{I}$$

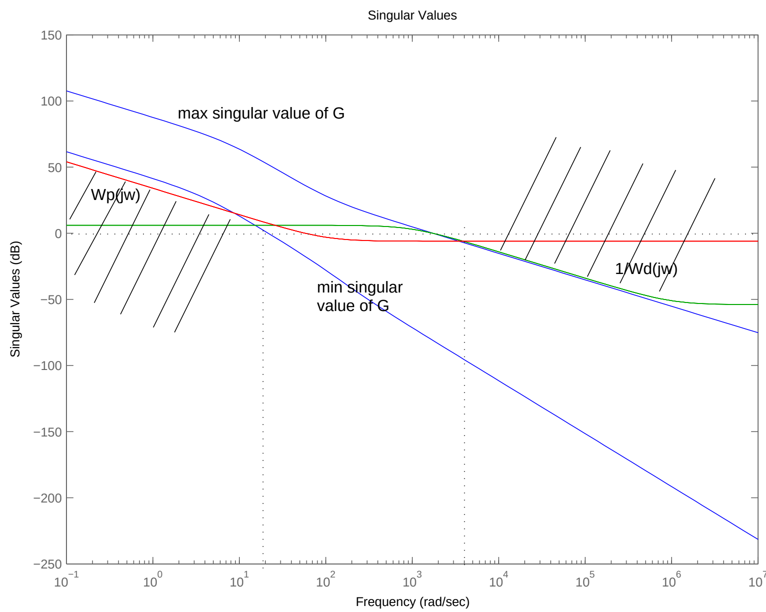
Problem is to find internally stabilizing controller, $\mathbf{K}$, such that

$$\overline{\sigma}(\mathbf{W}_p\mathbf{S}_o(j\omega)) + \overline{\sigma}(\mathbf{W}_\Delta\mathbf{T}_o(j\omega)) \leq 1$$

for all ω (solve the robust performance problem)

Graphical Loopshaping Solution

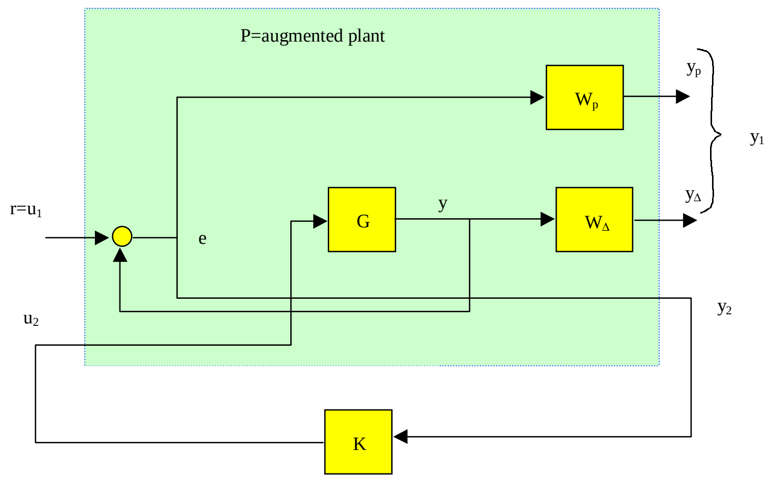
In this case we used the controller $\mathbf{K}(s) = \begin{bmatrix} \frac{120}{s} & 0 \\ 0 & \frac{120}{s} \end{bmatrix}$ to obtain the following loop shapes,



This solution, however, does not assure internal stability. In general it is impossible to use graphical methods to assure stability in MIMO loopshaping.

H_∞ Loopshaping

Consider the augmented plant shown below,



$$\|F_\ell(P,K)\|_\infty < \frac{1}{2} \Rightarrow \text{Robust Performance}$$

Matlab Synthesis Method

- Form system objects for plant, W_p, W_u, and W_Δ.
- Initially choose $W_u = 0$.
- Construct Augmented Plant `augss`
- Compute H_∞ controller, `hinf`
- Compute reduced order controller `schmr`
- Double Check System performance.

Initial Design Result

```
Solving Riccati equations and performing H-infinity
existence tests:
   1.  Is D11 small enough?                      OK
   2.  Solving state-feedback (P) Riccati ...
        a.  No Hamiltonian jw-axis roots?        FAIL
        b.  A-B2*F stable (P >= 0)?              FAIL
   3.  Solving output-injection (S) Riccati ...
        a.  No Hamiltonian jw-axis roots?        OK
        b.  A-G*C2 stable (S >= 0)?              OK
   4.  max eig(P*S) < 1 ?                        OK
-------------------------------------------------------
  NO STABILIZING CONTROLLER MEETS THE SPEC. !!
```

Set Control Weight W_u

Choose $W_u(s) = \varepsilon\mathbf{I}$ where $\varepsilon = .0001$ and redo.

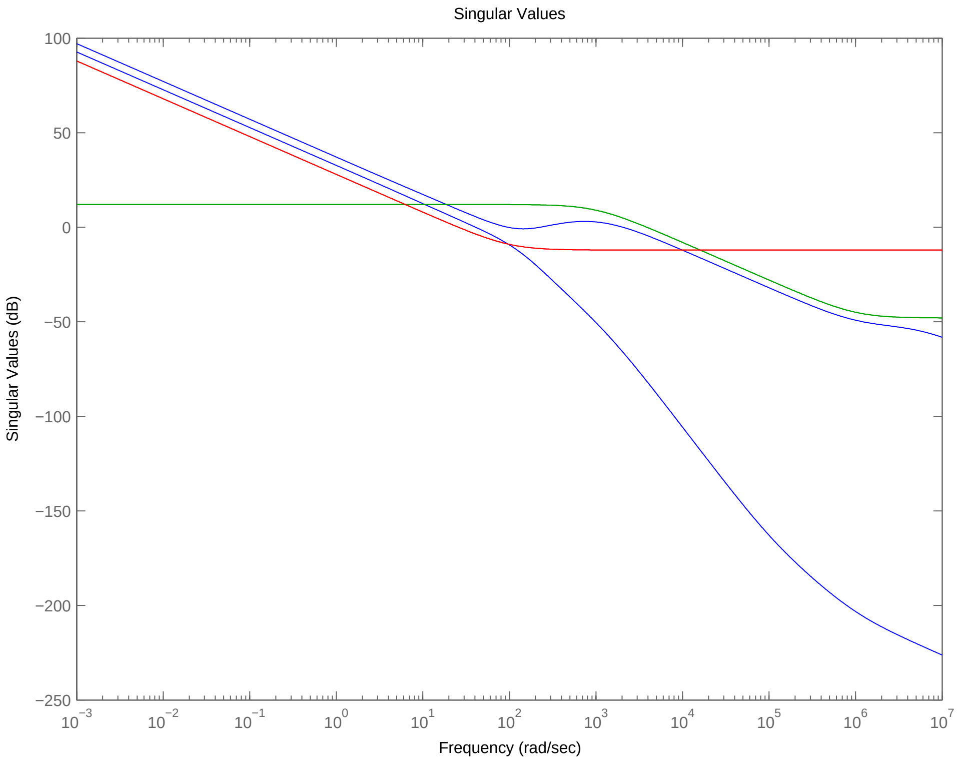
```
Solving for the H-inf controller F(s) using U(s) = 0 (default)
Solving Riccati equations and performing H-infinity
existence tests:
   1.  Is D11 small enough?                            OK
   2.  Solving state-feedback (P) Riccati ...
        a.  No Hamiltonian jw-axis roots?              OK
        b.  A-B2*F stable (P >= 0)?                    OK
   3.  Solving output-injection (S) Riccati ...
        a.  No Hamiltonian jw-axis roots?              FAIL
        b.  A-G*C2 stable (S >= 0)?                    FAIL
   4.  max eig(P*S) < 1 ?                              OK
-------------------------------------------------------
  NO STABILIZING CONTROLLER MEETS THE SPEC. !!
                    DONE!!!
```

Modify W_p

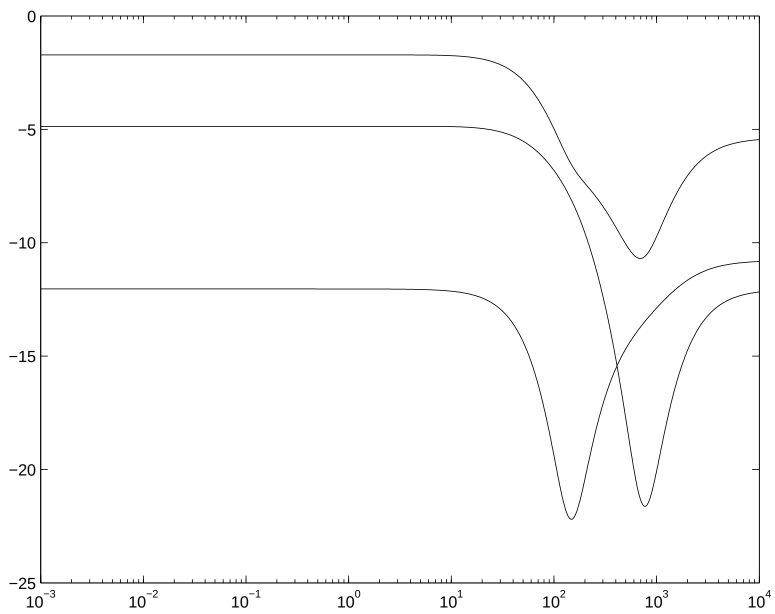
Previous design's augmented plant has a pole on the $j\omega$ axis. So choose $W_p(s) = \frac{s+100}{2s+\varepsilon}\mathbf{I}$ where $\varepsilon = .0001$.

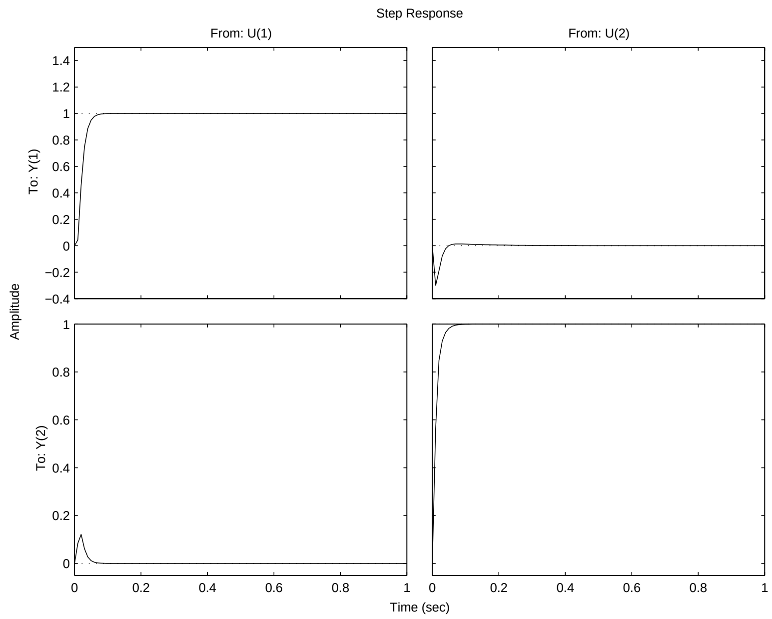
```
 existence tests:
    1.  Is D11 small enough?                       OK
    2.  Solving state-feedback (P) Riccati ...
         a.  No Hamiltonian jw-axis roots?         OK
         b.  A-B2*F stable (P >= 0)?               OK
    3.  Solving output-injection (S) Riccati ...
         a.  No Hamiltonian jw-axis roots?         OK
         b.  A-G*C2 stable (S >= 0)?               OK
    4.  max eig(P*S) < 1 ?                         OK
-------------------------------------------------------
   all tests passed -- computing H-inf controller ...
                     DONE!!!
-------------------------------------------------------
```

Loopshape of Final Design



Double Check





Controller Size

Note that the controller is of 8th order

```
>> Kss

a =
                 x1          x2          x3          x4          x5
   x2     3.975e+005  -1.977e+005  -1.312e+005   -1.08e+005  -7.681e+005
   x3     2.207e+005  -1.097e+005  -7.369e+004  -5.916e+004  -4.265e+005
   x4     1.107e+005  -5.515e+004  -3.585e+004   -3.08e+004  -2.139e+005
   x5     3.755e+005  -1.868e+005  -1.241e+005   -1.02e+005  -7.256e+005
   x6     1.011e+005  -5.029e+004  -3.341e+004  -2.745e+004  -1.954e+005
   x7     4.332e+004  -2.155e+004  -1.431e+004  -1.177e+004  -8.375e+004
   x8     2.874e+004  -1.429e+004        -9502        -7796  -5.551e+004

                 x6          x7          x8
```

x1	4.047e+006	4.317e+007	6.325e+007
x2	-1.337e+006	-1.426e+007	-2.09e+007
x3	-7.425e+005	-7.921e+006	-1.16e+007
x4	-3.723e+005	-3.972e+006	-5.819e+006
x5	-1.263e+006	-1.348e+007	-1.974e+007
x6	-3.402e+005	-3.629e+006	-5.316e+006
x7	-1.458e+005	-1.556e+006	-2.28e+006
x8	-9.663e+004	-1.032e+006	-1.513e+006

This is because the augmented plant is 8th order and it suggests we should be able to find a **reduced** size controller that will work also.

Controller Reduction

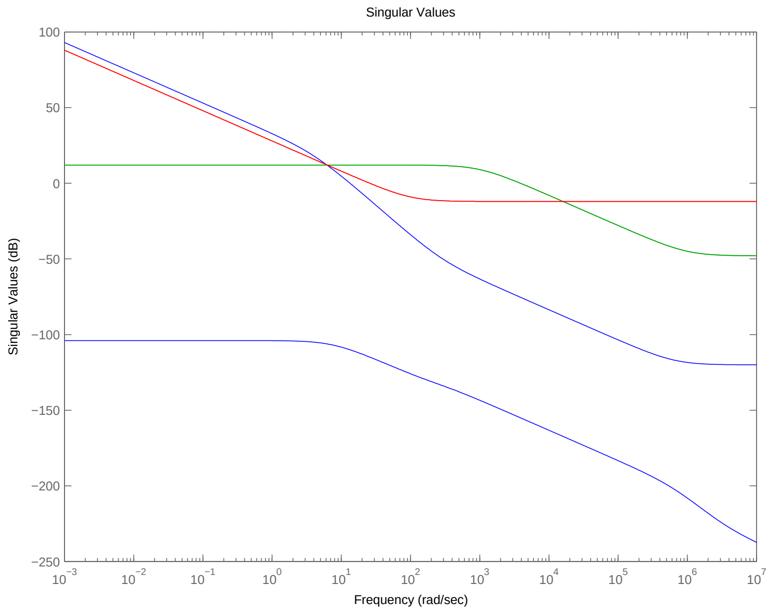
Given a system, G, the model reduction problem identifies another transfer function $\hat{G}$ such that

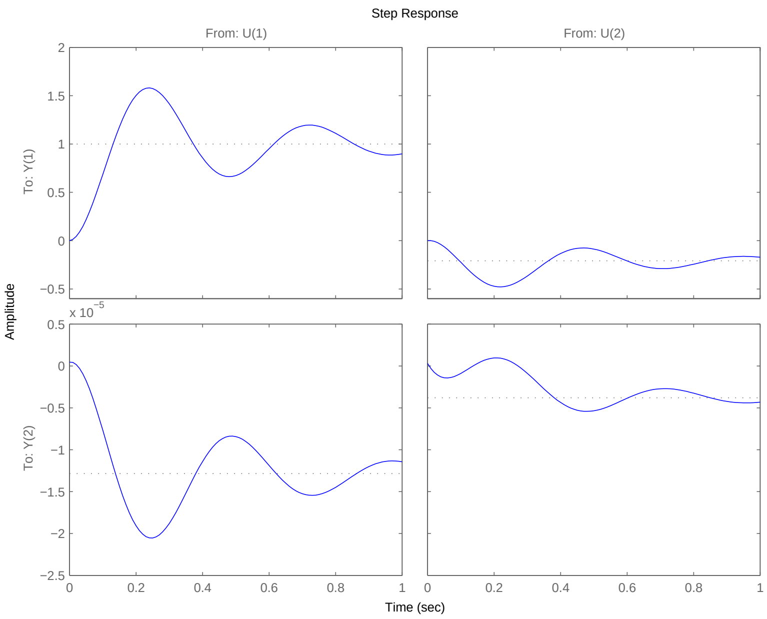
$$\|G - \hat{G}\|_\infty < \gamma < 1$$

and the order of $\hat{G}$ is less than the order of G.

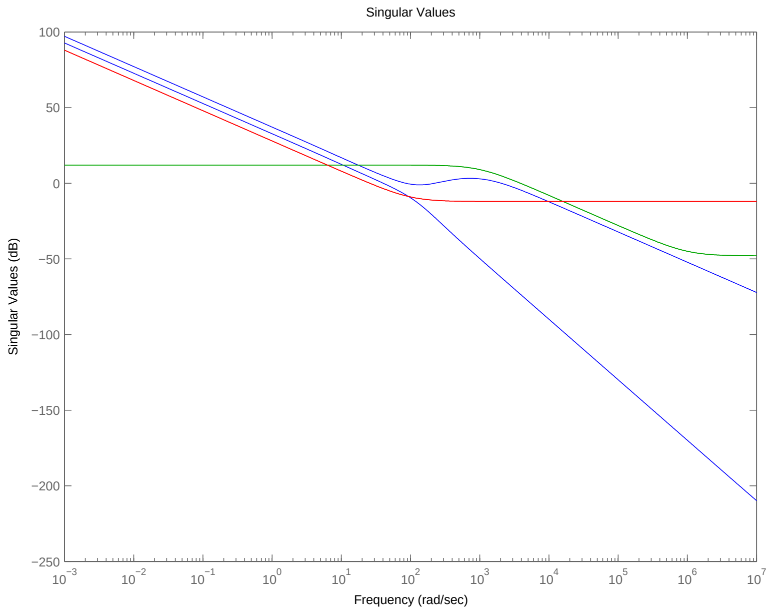
Matlab provides a function `schmr` to perform model reduction.

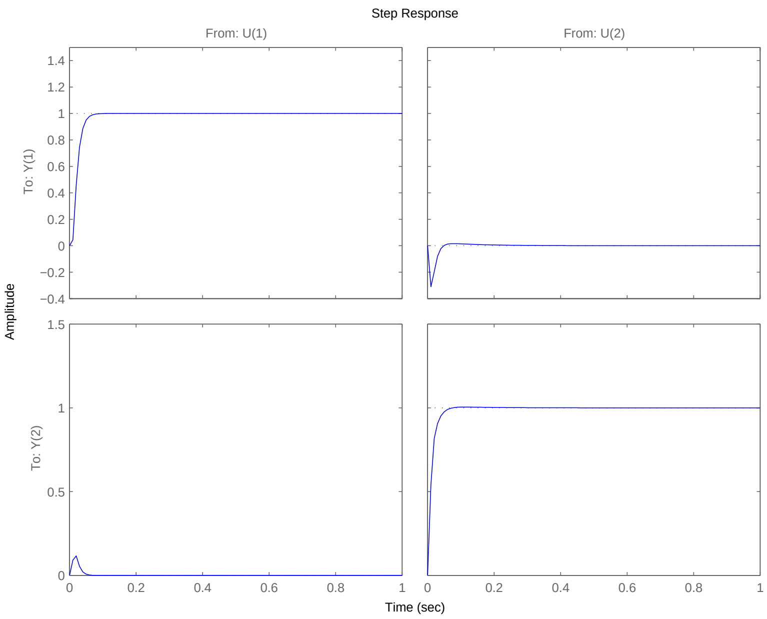
1st Order Controller Loopshape





4th Order Controller Loopshape





Future Topics

- Balanced Model Realizations
- Balanced Model Reductions
- Three Simplified Generalized Regulator Problems
- Output Feedback Regulator Problem
- Algebraic Riccati Equation