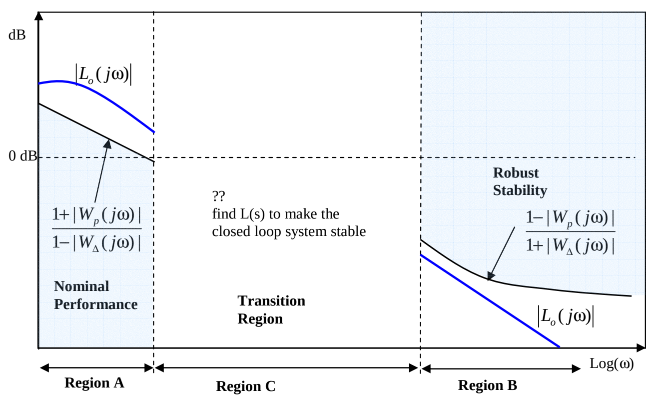
Loopshaping Conditions for Scalar Systems

- Low frequency condition (frequencies where $|L_o(j\omega)| > 1$)

$$|L_o(j\omega)| > \frac{1+|W_p(j\omega)|}{1-|W_\Delta(j\omega)|} \Rightarrow |W_p(j\omega)S_o(j\omega)| + |W_\Delta(j\omega)T_o(j\omega)| < 1$$

- High frequency condition (frequencies where $|L_o(j\omega)| < 1$)

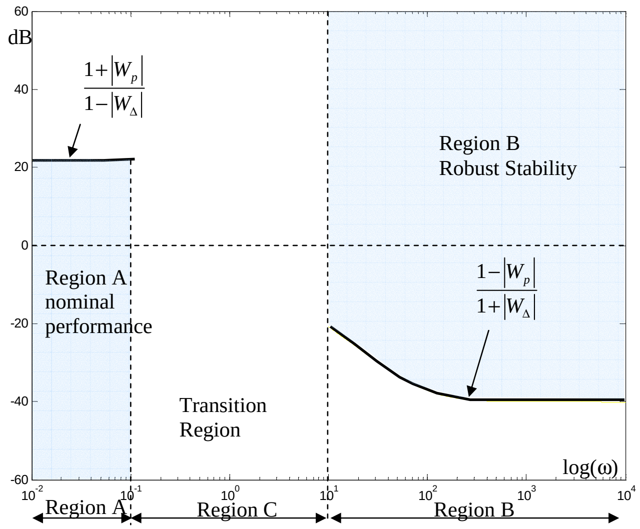
$$|L_o(j\omega)| < \frac{1-|W_\Delta(j\omega)|}{1+|W_p(j\omega)|} \Rightarrow |W_p(j\omega)S_o(j\omega)| + |W_\Delta(j\omega)T_o(j\omega)| < 1$$

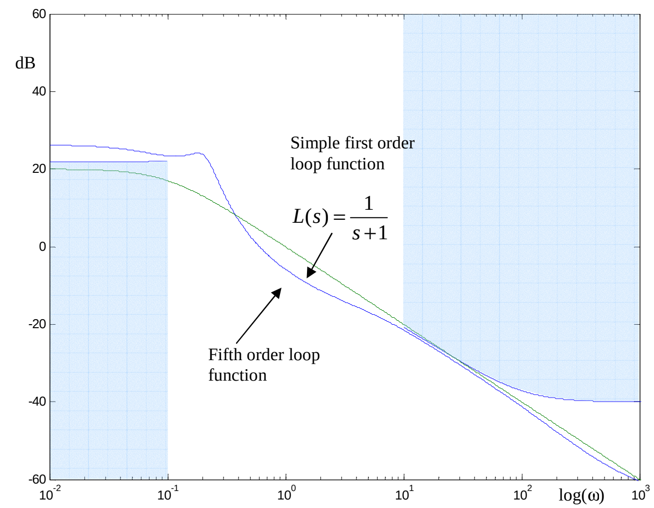


Does not ensure internal stability!

Example: desirable loop function

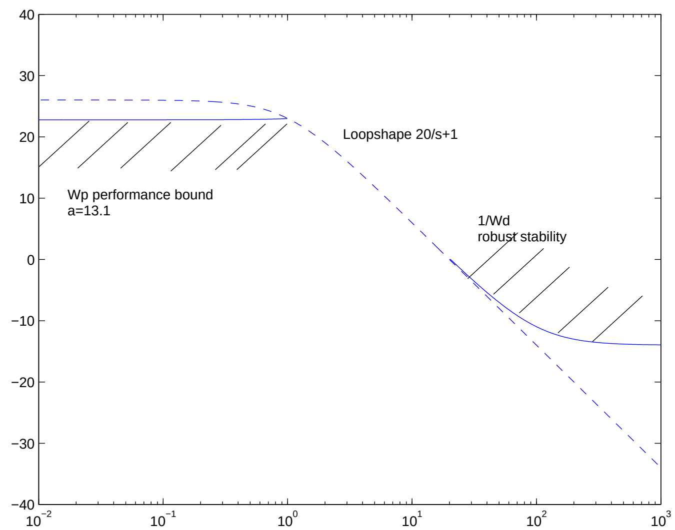
- Plant $G(s) = \frac{1}{s(s+1)}$
- Low Frequency Objective, $|W_p(j\omega)| < \begin{cases} 10 & 0 < \omega < 0.1 \\ 0 & \text{otherwise} \end{cases}$
- High Frequency Objective, $W_\Delta(s) = 100\frac{s+0.1}{s+100}$.
- Desired Loop $L(s) = \frac{1}{s+1}$ or use a higher order system obtained using `fitmag`

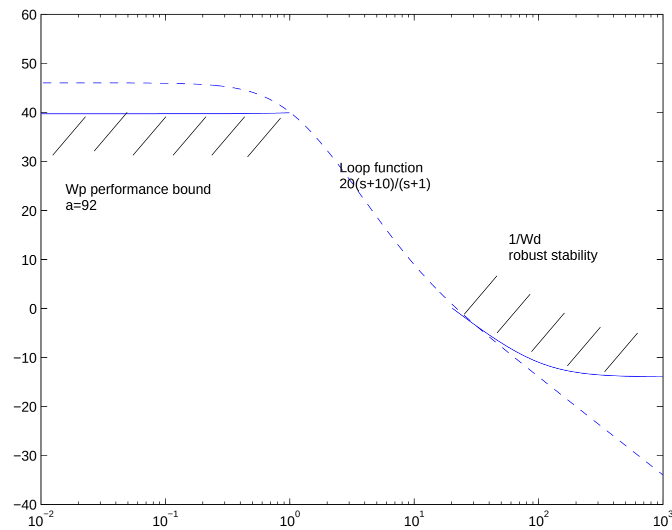




Example: Proportional and Lag Compensation

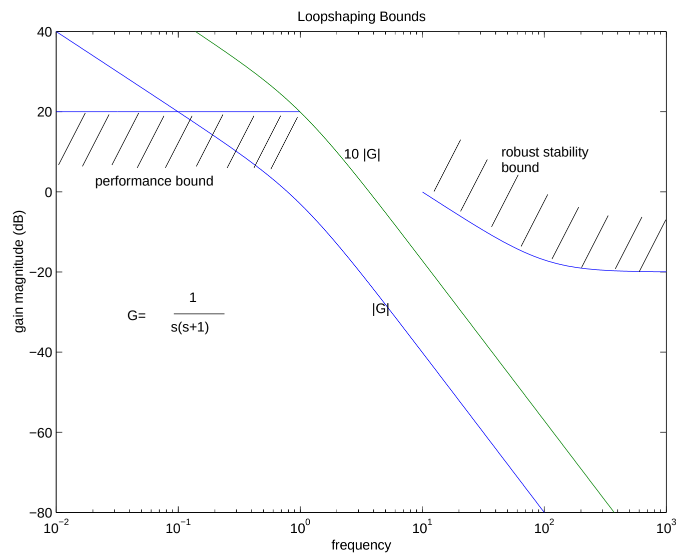
- Plant $L(s) = \frac{k}{s+1}$
- Low frequency objective $|W_p(j\omega)| < \begin{cases} \gamma & \text{if } 0 \leq \omega < 1 \\ 0 & \text{otherwise} \end{cases}$
- High frequency objective $W_\Delta(s) = \frac{s+1}{20(0.01s+1)}$.
- Find $k = 20$ that gives "largest" $\gamma = 13.1$
- Use "lag" compensator $k(s) = 20\frac{s+10}{s+1}$ to obtain $\gamma = 92$.

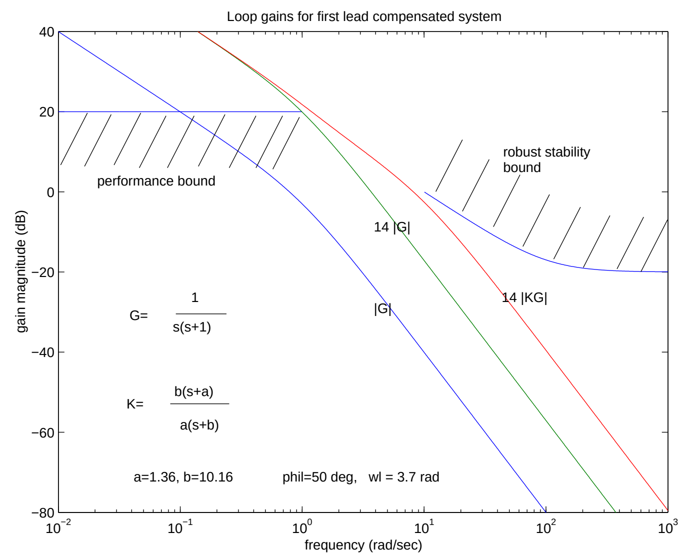




Example: Lead Compensation

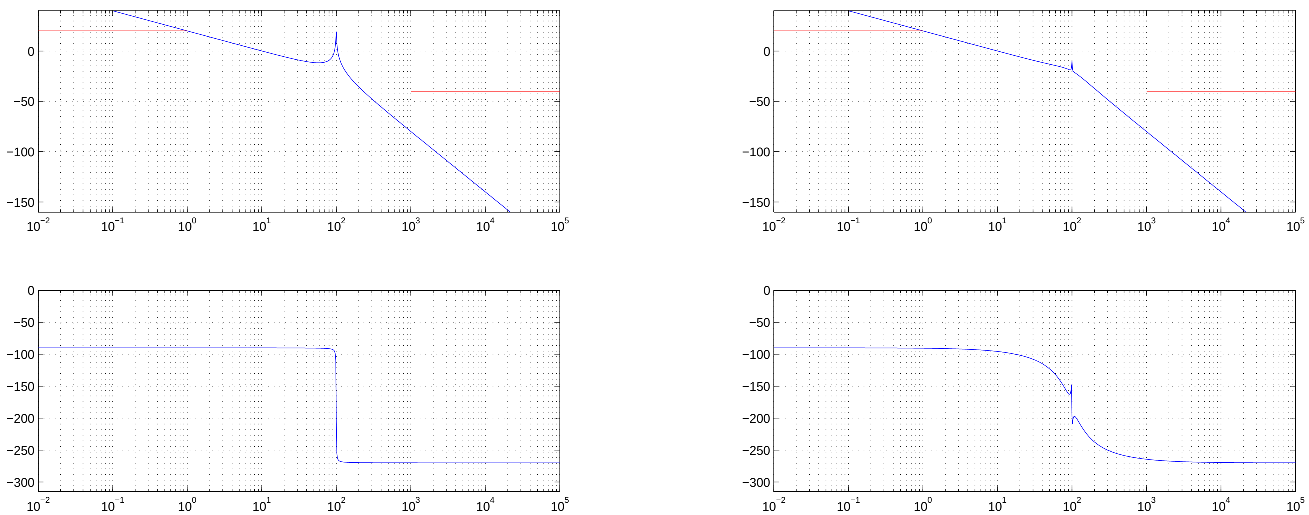
- Plant $G(s) = \frac{1}{s(s+1)}$
- Low frequency objective $|W_p(j\omega)| = \begin{cases} 10 & 0 \leq \omega < 1 \\ 0 & \text{otherwise} \end{cases}$
- High frequency objective $W_\Delta(s) = \frac{s+1}{10(0.01s+1)}$
- Lead Compensator to assure 60° of phase crossover, $K(s) = 14\frac{(s/1.36)+1}{(s/10.16)+1}$





Example:Notch Filtering

- Plant $G(s) = \frac{1.e5}{s(s^2+s+1.e4)}$
- Low Frequency Objective $|W_p(j\omega)| = \begin{cases} 10 & 0 \leq \omega < 0.1 \\ 0 & \text{otherwise} \end{cases}$
- High Frequency Objective $W_\Delta(s) = \begin{cases} 100 & \omega > 1000 \\ 0 & 0 < \omega < 1000 \end{cases}$
- Compensator $K(s) = \frac{s^2+2\eta_n\omega_n s+\omega_n^2}{s^2+\omega_n s+\omega_n^2}$



Common Loopshaping Blocks

- Proportional Gain
- Lag Compensator (PI controller)
- Lead Compensator (PD controller)
- Lead-Lag Compensator (PID controller)
- Notch Filter (suppress resonant mode)
- High-Gain Rolloff (increase high frequency attenuation)