HIMAT Example

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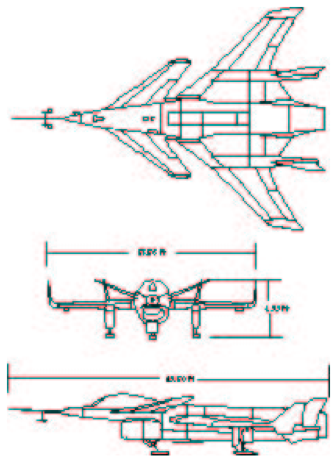
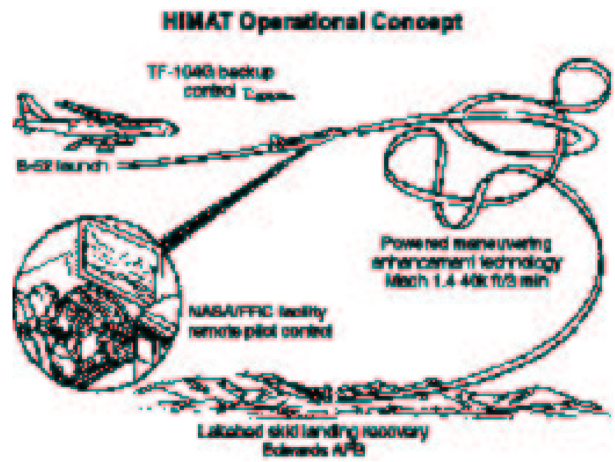
Abstract: This lecture illustrates the use of $\mathcal{H}_\infty$ loopshaping on the HIMAT system. Also introduces $\mathcal{H}_\infty$ loopshaping for plants with coprime-factored uncertainties.

HIMAT Vehicle

The HIMAT vehicle is a scaled remotely piloted vehicle (RPV) of an advanced fighter that was flight tested in the late 1970's.



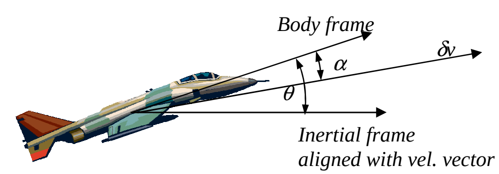


Problem Statement

State Variables	Control Variables	Measurements
δv - velocity perturbations	δ_e elevon angle	α angle of attack
α - angle of attack	δ_c canard angle	θ pitch angle
q - pitch rate		
θ - pitch angle		

A linearized model for the plane's longitudinal dynamics is:

$$\mathbf{G}_0 \stackrel{s}{=} \left[\begin{array}{cccc|cc} -.023 & -37 & -19 & -32 & 0 & 0 \\ 0 & -1.9 & .98 & 0 & -.41 & 0 \\ .012 & -12 & -2.6 & 0 & -78 & 22 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 57 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 57 & 0 & 0 \end{array}\right]$$



Model Uncertainty

An unstructured multiplicative uncertainty model for the aircraft is

$$\mathbf{G} = \mathbf{G}_0(\mathbf{I} + \Delta)$$

where Δ is the uncertain part of the system. We assume Δ is stable such that $\overline{\sigma}(\mathbf{W}_\Delta^{-1}(j\omega)\Delta) \leq 1$ for all ω where

$$W_\Delta(s) = \begin{bmatrix} \frac{50(s+100)}{s+10000} & 0 \\ 0 & \frac{50(s+100)}{s+10000} \end{bmatrix}$$

Such an uncertainty bound is assumed to have been obtained from exhaustive empirical testing of the aircraft.

Performance Specification

The performance of the closed loop system will be evaluated by the output sensitivity function. In particular, we assume the following performance specification

$$\overline{\sigma}(\mathbf{S}(j\omega))\mathbf{W}_p(j\omega) \leq 1$$

for all ω and the performance weighting function is

$$\mathbf{W}_p(s) = \begin{bmatrix} \frac{0.5(s+\beta)}{s+0.03} & 0 \\ 0 & \frac{0.5(s+\beta)}{s+0.03} \end{bmatrix}$$

where β is a constant that we'll specify in a second. Such a performance weight indicates that at low frequencies (frequencies below β) the closed loop system should reject disturbances at the output by a factor of $50\beta/3$. At high frequencies, the performance gets less stringent. The parameter β, therefore, characterizes some important qualitative properties of the closed loop system. The larger β, the better this system.

Control Weighting Function

We need to make sure that the control signals are weighted with $\mathbf{W}_u = \varepsilon\mathbf{I}$.

The following script computes the reduced order controller for this problem.

```
norder=9;
[A,B1,B2,C1,C2,D11,D12,D21,D22]=...
    augss(Ag,Bg,Cg,Dg,...
          Ap,Bp,Cp,Dp,...
          Au,Bu,Cu,Du,...
          Ad,Bd,Cd,Dd)
[acp,bcp,ccp,dcp,acl,bcl,ccl,dcl]=...
    hinf(A,B1,B2,C1,C2,D11,D12,D21,D22);
[am,bm,cm,dm,totbnd,hsv]=...
    schmr(acp,bcp,ccp,dcp,1,norder);
```

Uncompensated System $\beta = 3$

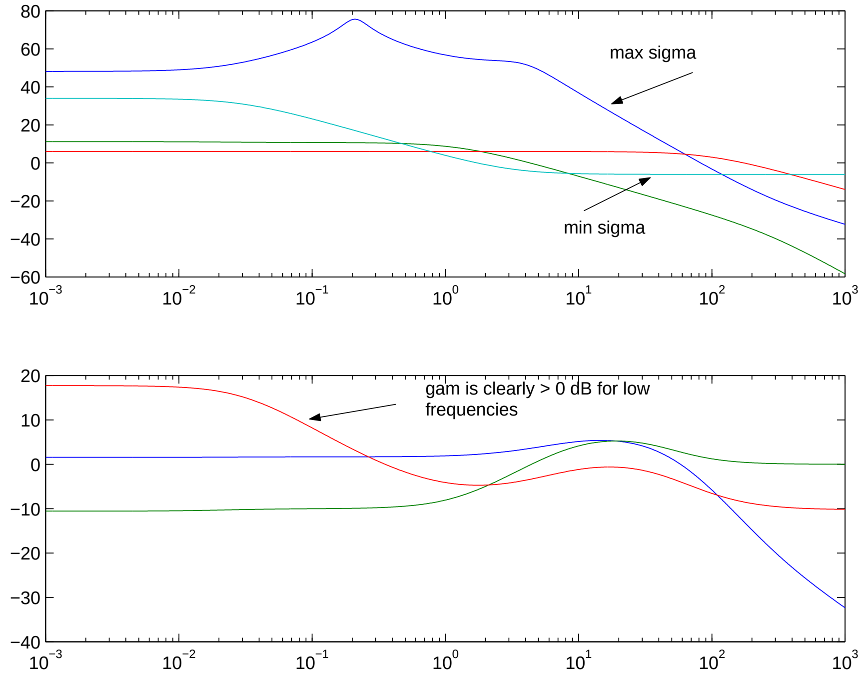


Figure 1: (top) Uncompensated plant's singular value plot, $\beta = 3$. (bottom) singular values $\mathbf{S}_o$, $\mathbf{T}_o$, and $\overline{\sigma}(W_p S_0) + \overline{\sigma}(W_\Delta T_0)$

Compensated System Robust Performance $\beta = 3$

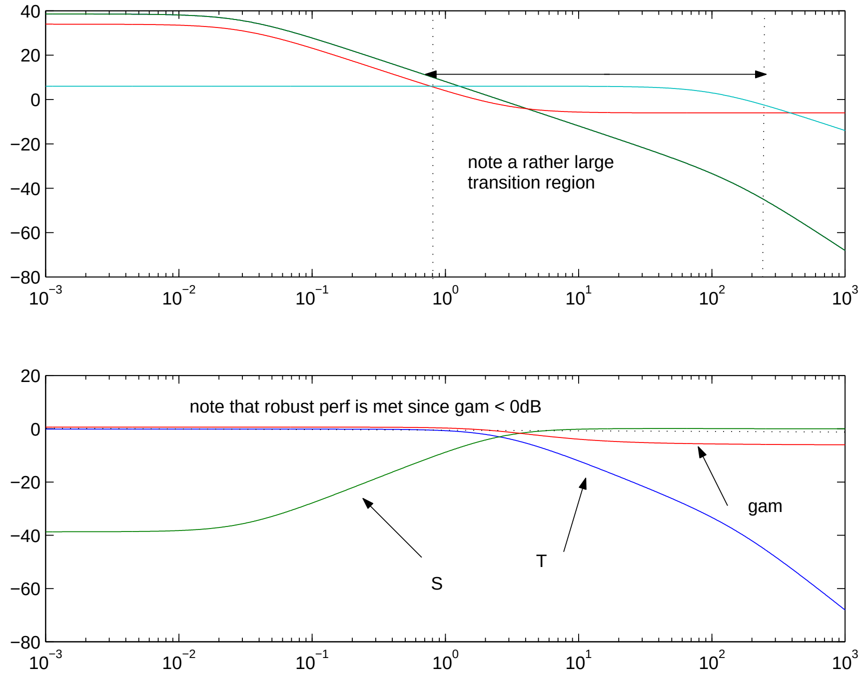
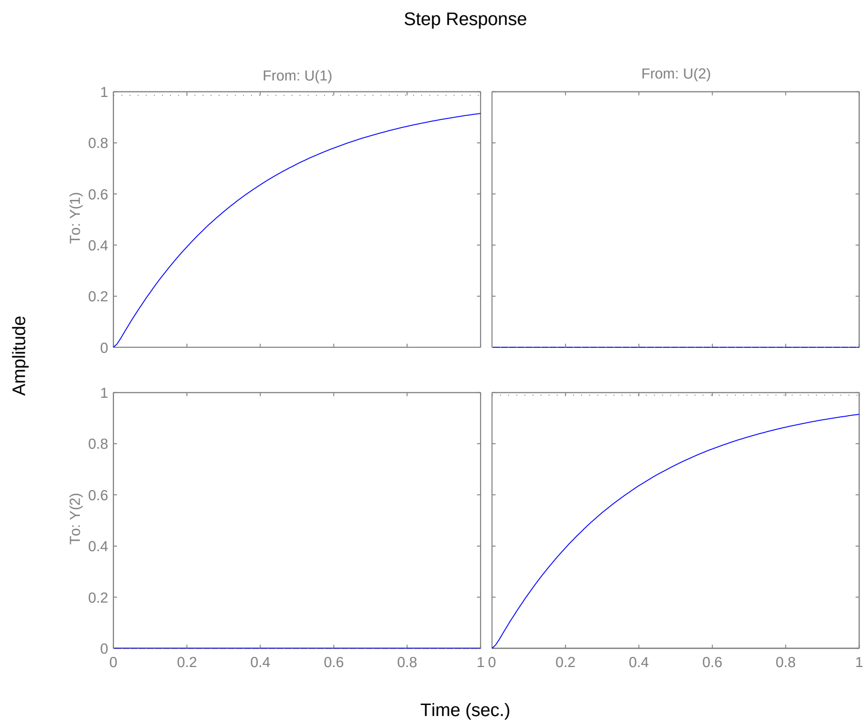


Figure 2: (top) compensated plant's singular value plot, $\beta = 3$. (bottom) singular values of $\mathbf{S}_o$, $\mathbf{T}_o$, and $\overline{\sigma}(W_p S_0) + \overline{\sigma}(W_\Delta T_0)$

Compensated System's Step Response

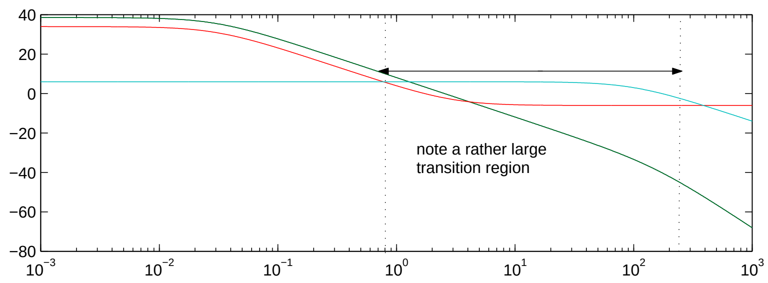
We finalize this design by plotting the step responses for the closed loop system.

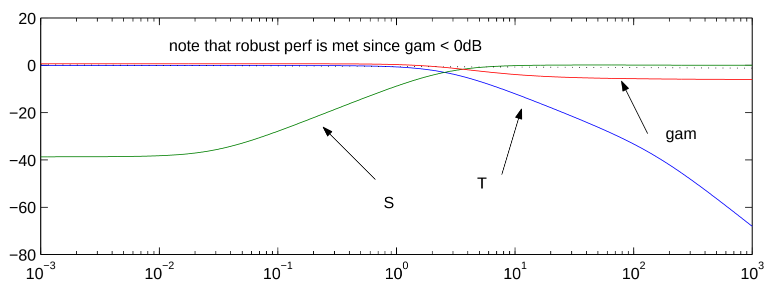


The step responses show that the system has a rise time of 1.0 seconds. This is rather slow for a high performance fighter.

Speeding Up Response

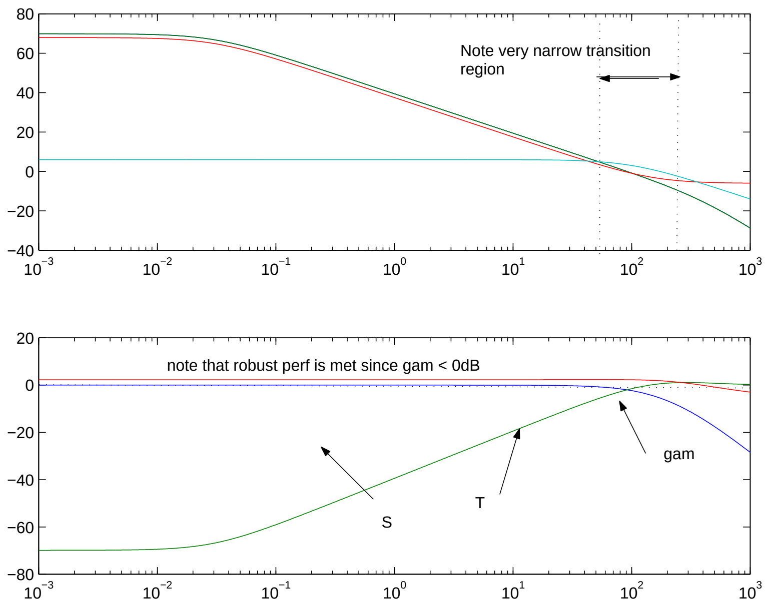
Note that the bandwidth of the closed loop system is determined by the width of the transition region. The lower end of this region is fixed by β, so we expect that by increasing β, we can speed up the the system's response.

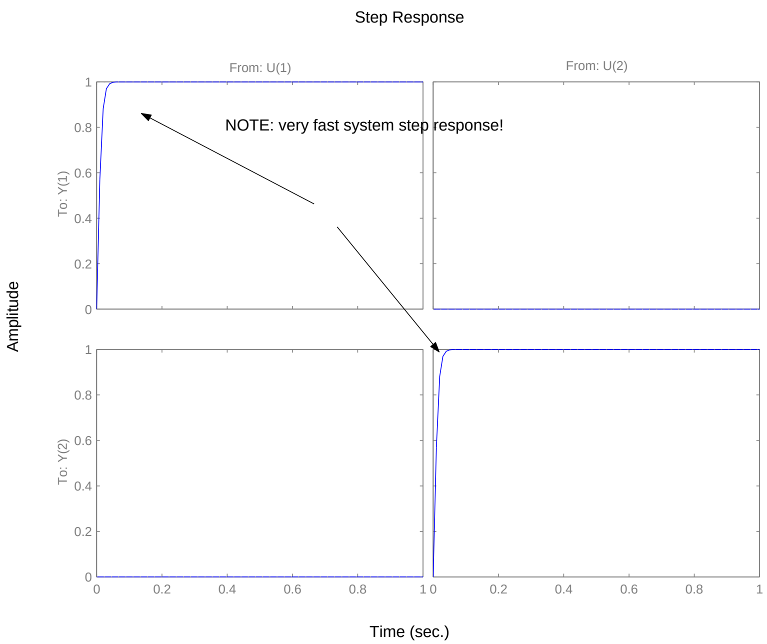




If β is too large then it is inconsistent with the uncertainty bound $\mathbf{W}_{\Delta}^{-1}(s)$. From our plots, we expect this to happen around $100 - 200$ rad/sec.

Redesign for speed $\beta = 150$





Note that the rise time of the redesigned system is 20 msec. Much faster

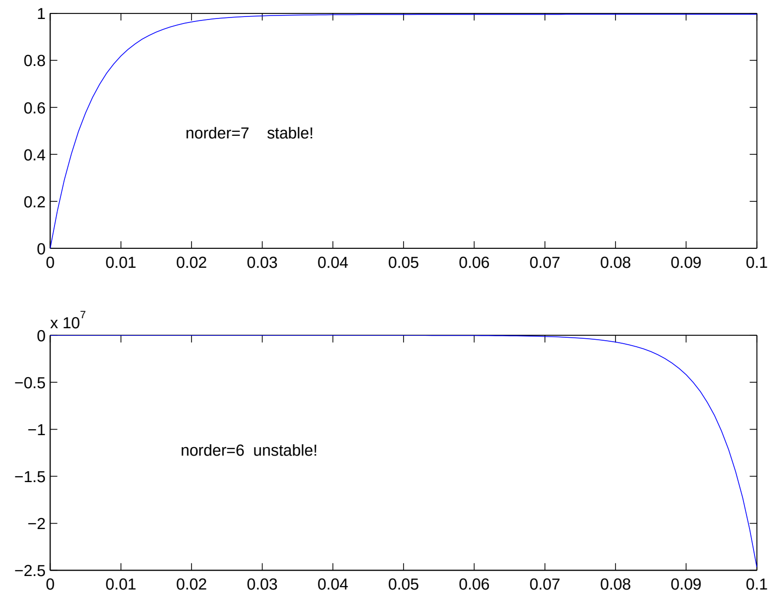
Upper Limit on Performance

The upper limit on performance occurs for $\beta = 300$. At $\beta = 310$, we get:

```
          << H-inf Optimal Control Synthesis >>
 Solving Riccati equations and performing H-infinity
 existence tests:
    1.  Is D11 small enough?                         OK
    2.  Solving state-feedback (P) Riccati ...
         a.  No Hamiltonian jw-axis roots?           OK
         b.  A-B2*F stable (P >= 0)?                 FAIL
    3.  Solving output-injection (S) Riccati ...
         a.  No Hamiltonian jw-axis roots?           OK
         b.  A-G*C2 stable (S >= 0)?                 OK
    4.  max eig(P*S) < 1 ?                           OK
-------------------------------------------------------
   NO STABILIZING CONTROLLER MEETS THE SPEC. !!
           -- CLOSED-LOOP UNSTABLE --
```

Model Reduction

Recall that $\mathcal{H}_\infty$ controllers are of extremely high order because the compensator order equals the order of the augmented plant. We use Schur balanced model reduction to obtain a lower order controller.



Coprime Factored Uncertainty

The preceding example assumed an unstructured multiplicative uncertainty on the plant's output.

A more general uncertainty model is the coprime factored uncertainty model,

$$\mathbf{G} \in (M+\Delta_M)^{-1}(N+\Delta_N)$$

where (M,N) is a left coprime factorization of $\mathbf{G}_o$ over $\mathcal{R}H_\infty$. The uncertainty factors, Δ_M and Δ_N, are in $\mathcal{R}H_\infty$ and

$$\left\| \begin{bmatrix} \Delta_N & \Delta_M \end{bmatrix} \right\|_\infty < \varepsilon$$

This system is robustly stable (robust performance) if and only if

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} \left(I + G_o K \right)^{-1} M^{-1} \right) \right\|_\infty \leq 1/\varepsilon$$

$\mathcal{H}_\infty$ Loopshaping (coprime factored uncertainties)

Chapter 16 in Big Blue discusses $\mathcal{H}_\infty$ loopshaping for plants with coprime factored uncertainties.

1. **Loopshaping**: Use W_1 and W_2 to form desired open loop function $G_s = W_2 G_o W_1$. Compute the normalized left coprime factorization of G_s over $\mathcal{R}H_\infty$ (i.e. $G_s = M_s^{-1} N_s$).
2. **Robust Stabilization**: Calculate $\varepsilon_{\max}$ where

$$\varepsilon_{\max} = \sqrt{1 - \left\| \begin{bmatrix} N_s & M_s \end{bmatrix} \right\|_H^2} < 1$$

   if $\varepsilon_{\max} < 1$ readjust W_1 and W_2 and repeat.

3. **Controller Design**: Select $\varepsilon < \varepsilon_{\max}$ and find the internally stabilizing $\mathcal{H}_\infty$ controller, K_∞ such that

$$\left\| \begin{bmatrix} K_\infty \\ I \end{bmatrix} \left(I + G_s K_\infty \right)^{-1} M_s^{-1} \right) \right\|_\infty \leq 1/\varepsilon$$

   Final controller $K = W_1 K_\infty W_2$