Higher Order Networks & BuildHon+

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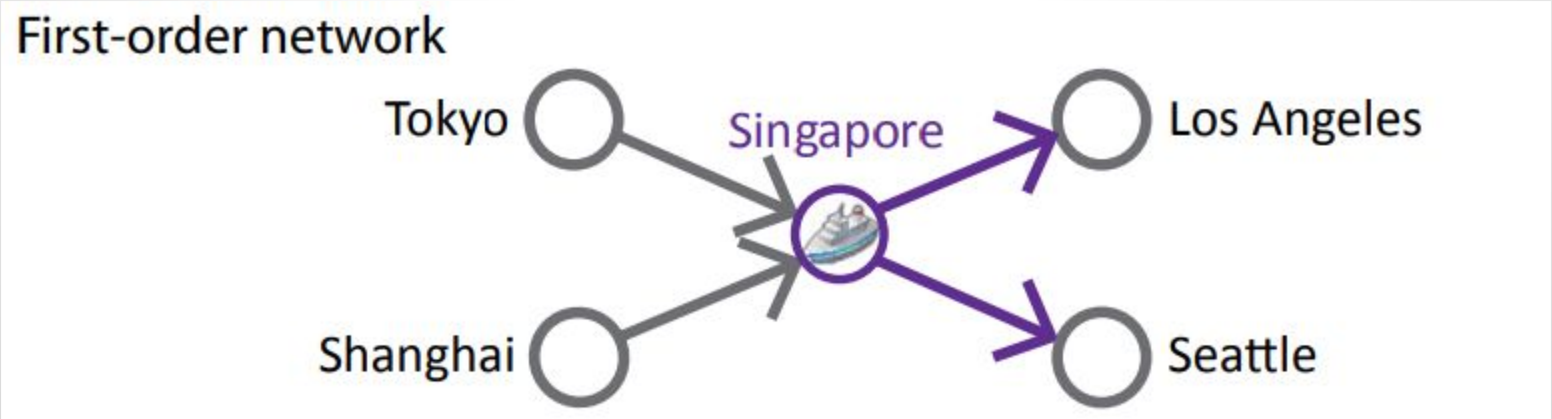
The Problem

How do we **represent** big data as a network, while accurately preserving dependencies?



A Solution!

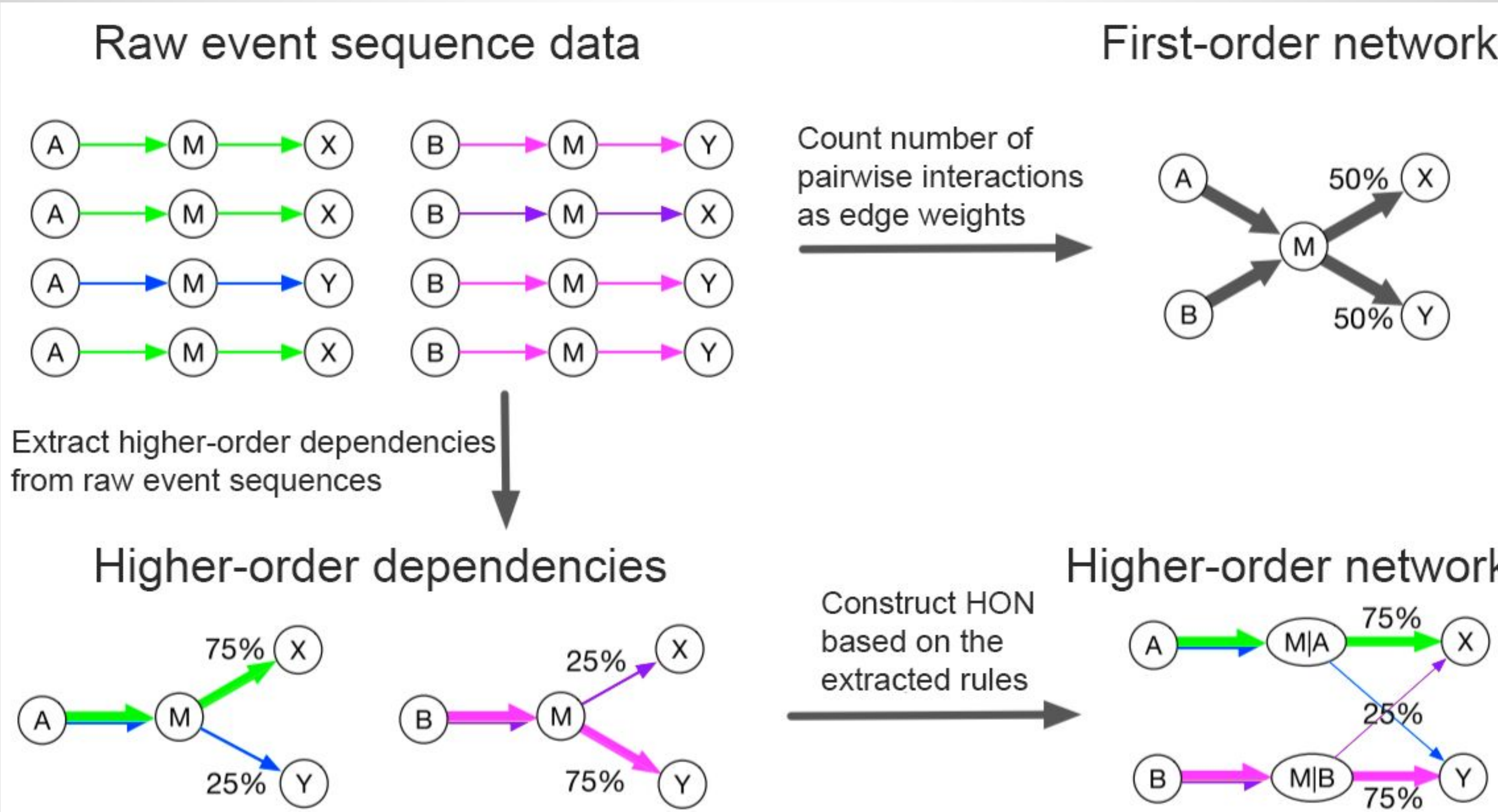


Image from [2]

The Kernel: BuildHon

Algorithm used to construct the HON

Has 2 main steps:

1. Rule extraction
2. Network rewiring

Step 1: Rule Extraction

```
cur_ord = 1;
seqs = get_raw_sequences();
first_order = build_observations(seqs, cur_ord);
rules.append(first_order);

while (rules.last != empty AND current_order < MAX_ORDER) {
    next_cands = get_next_order_candidates(rules.last);
    next_ord_obs = build_observations(seqs, cur_ord, next_cands);
    next_rule = check_and_extend(rules.last, next_obs);
    rules.append(next_rule);
}
```

Step 1: Rule Extraction

1. Count the number of sequential node interactions at the first-order (basically the normal network)
2. Normalize the distributions for each pairwise interaction
3. For each fork node, add the preceding step and see how that changes the distribution of the sequence
4. If the change is “significant” (above a selected threshold), add a second-order dependency and repeat the process recursively to determine higher orders

Step 2: Rewire the Network

…

Time Complexity

$$L * N * \sum_{i=1}^{k} ((i + 1)\, R_i)$$

where L is the count of records in the raw data;
N is the number of unique nodes in the raw data;
k is the maximum order of dependency;
R_i is the count of dependencies at order i

(*the theoretical upper bound is exponential but is not really helpful for real data sets, in which orders of dependency tend to follow an inverse power law)

Data Sets

- Synthetic web clickstreams (11 billion nodes)
  - Subsets with 1, 5, 10, 100 million nodes
- Global shipping data (3,415,577 voyages made by 65,591 ships between May 1st , 2012 and April 30th, 2013)

Implementation

- C++ implementation of Rule Extraction algorithm
- ~400 lines, not including header files & definitions
- (Close to the same as the original Python implementation, minus a couple utilities)
- Mostly vectors for fast iteration, plus one unordered_map (hash table)

Modules

build_observations(seqs, cur_ord, next_candidates=None):

```
for each c : next_candidates:
    // get all sequences of length cur_ord with target c
    // count all sequences and calculate out degree distribution
```

get_next_order_candidates(rules_base_order):

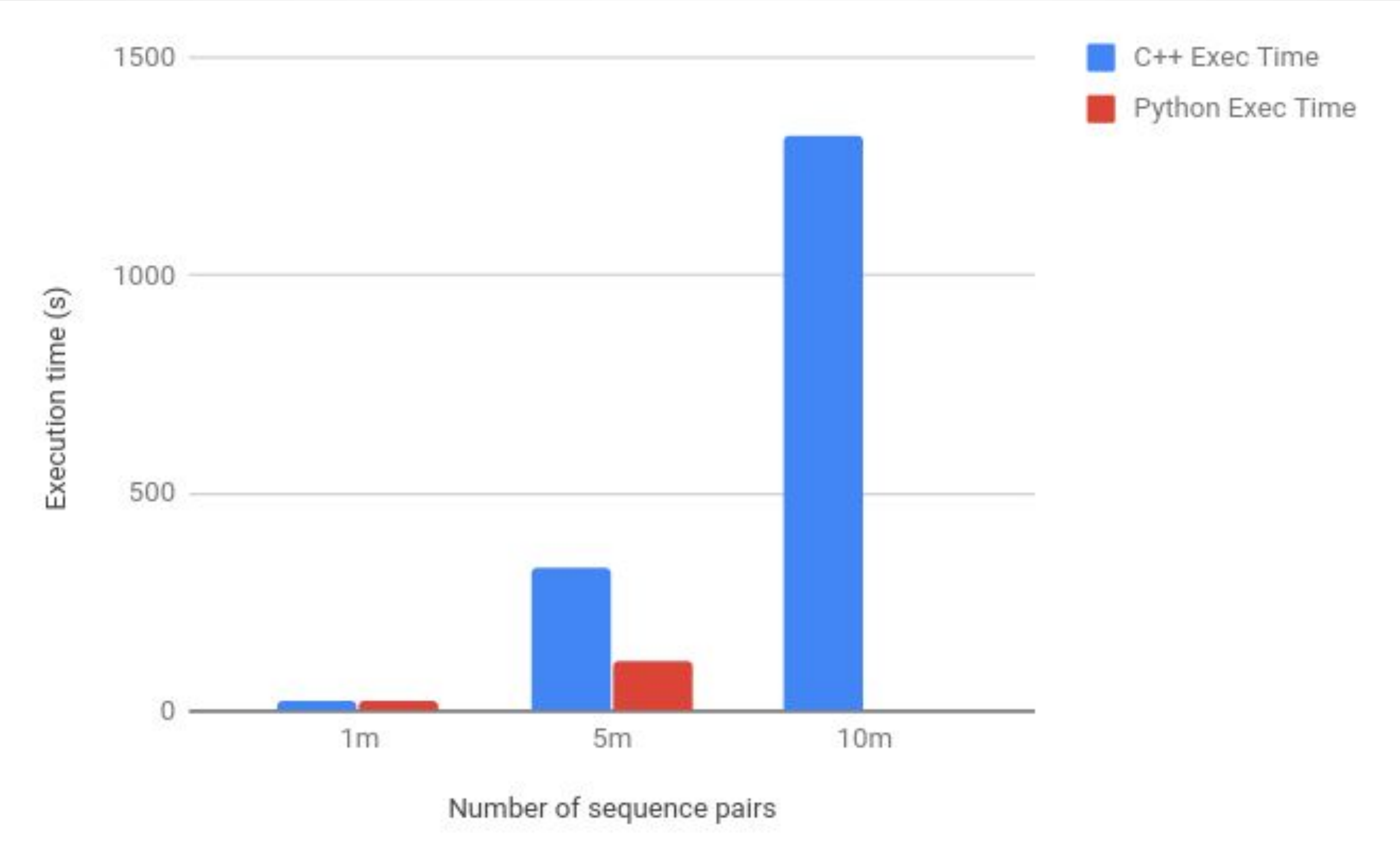
```
for each r : rules:
    if r.confidence < THRESHOLD:
        next_order_candidates.append(r)
```

Modules

```
check_and_extend(rules_base_order, next_order_obs):
for each r : rules_base_order:
    // get all candidate extensions from next_order_obs
    if (get_ext_significance(r, candidates) > EXT_THRESHOLD) {
        next_order_rules.append(candidates);
    }
return new_order_rules;
```

Initial Results

Number of Seq Pairs	C++ Exec Time (s)	C++ # Rules	Python Exec Time (s)	Python # Rules
1m	22.32	440	26.01	212
5m	328.52	2,200	116.12	1160
10m	1321.26	4400	-- (crashed)	--


1500
1000
500
0
Execution time (s)
1m
5m
10m
Number of sequence pairs
C++ Exec Time
Python Exec Time

Future Work

- Scrap the Rule Extraction? :-(
- Possible parallel implementation of Network Rewiring
  - Giraph or Stinger?
  - Need to think more through use case
- Possible method for validating network representation

- [1] Xu, J., Wickramarathne, T., & Chawla, N. (2016). Representing higher-order dependencies in networks. Science Advances, 2(5), E1600028.
- [2] http://www.higherordernetwork.com/
- [3] https://github.com/xyjprc/hon
- [4] Xu, J., Saebi, M., Ribeiro, B., Kaplan, L., & Chawla, N. (2017). Detecting Anomalies in Sequential Data with Higher-order Networks.
- [5] Cui Jiao, Guo Jun, Zhang Cangsong, & Chang Xiaojun. (2012). Implementation of random walk algorithm by parallel computing. Fuzzy Systems and Knowledge Discovery (FSKD), 2012 9th International Conference on, 2477-2481.
- [6] Fournier-Viger, P., Nkambou, R., & Tseng, V. S. M. (2011, March). RuleGrowth: mining sequential rules common to several sequences by pattern-growth. In Proceedings of the 2011 ACM symposium on applied computing (pp. 956-961). ACM.