

pp. 202. **The Universal Turing Machine** (Sec. 4.2)

- Remember $A_{TM} = \{ \langle M, w \rangle \mid M \text{ a TM that accepts } w \}$
- **KEY:** we can write a recognizer U, *but not a decider*
- Let's design a **Universal TM U** which *recognizes* A_{TM}
 - Input is $\langle M, w \rangle$
 - $\langle M \rangle$ is an "encoding" of M
 - $\langle w \rangle$ is an encoding of a string to be given to M
 - U *simulates* M processing w
 - If M ever enters its accept state, then U accepts,
 - If M ever enters its reject state, then U rejects
 - Note this loops on $\langle M, w \rangle$ if M loops on w
 - That's why it is NOT a decider
 - If we could determine if M never halts on w then U could halt in reject. That would make U a decider.

- What does U look like?
 - Long history of trying for “smallest” description
 - Typically defined in terms of “(m,n)”
 - m = # of states
 - n = # of tape symbols
- Turing’s original 1936 machine
 - <http://onlinelibrary.wiley.com/doi/10.1112/plms/s2-43.6.544/epdf>
 - Turing used 7 symbols to encode M { A, C, D, R, L, N, ; }
 - M’s State q_i encoded as DA^i
 - If M’s $\Sigma = \{a_0, a_1, \dots, a_k\}$, then U’s encoding of $a_i = DC^i$
 - a_0 is the blank
 - If M has $\delta(q_i, a_j) = (q_k, a_l, L)$ (similar for R)
 - Then encode as ; $DA^iDC^jDC^lLDA^k$
 - Actually code spread out on every other tape cell
 - Termed the “F-cells” which are never changed
 - With intermediate “E-cells” used to “tag” where M is
 - Similar to the “*” we used earlier
 - Last transition has a “;;” following it
 - Additional symbols {e, ::, u, v, x, y, z} written into E-cells
 - Processing:
 - Find transition that corresponds to current state and w_i
 - Update w_i and mark next state. Test for accept/reject

- From: Neary, Turlough; Woods, Damien (2009), "Four Small Universal Turing Machines", Fundamenta Informaticae, 91

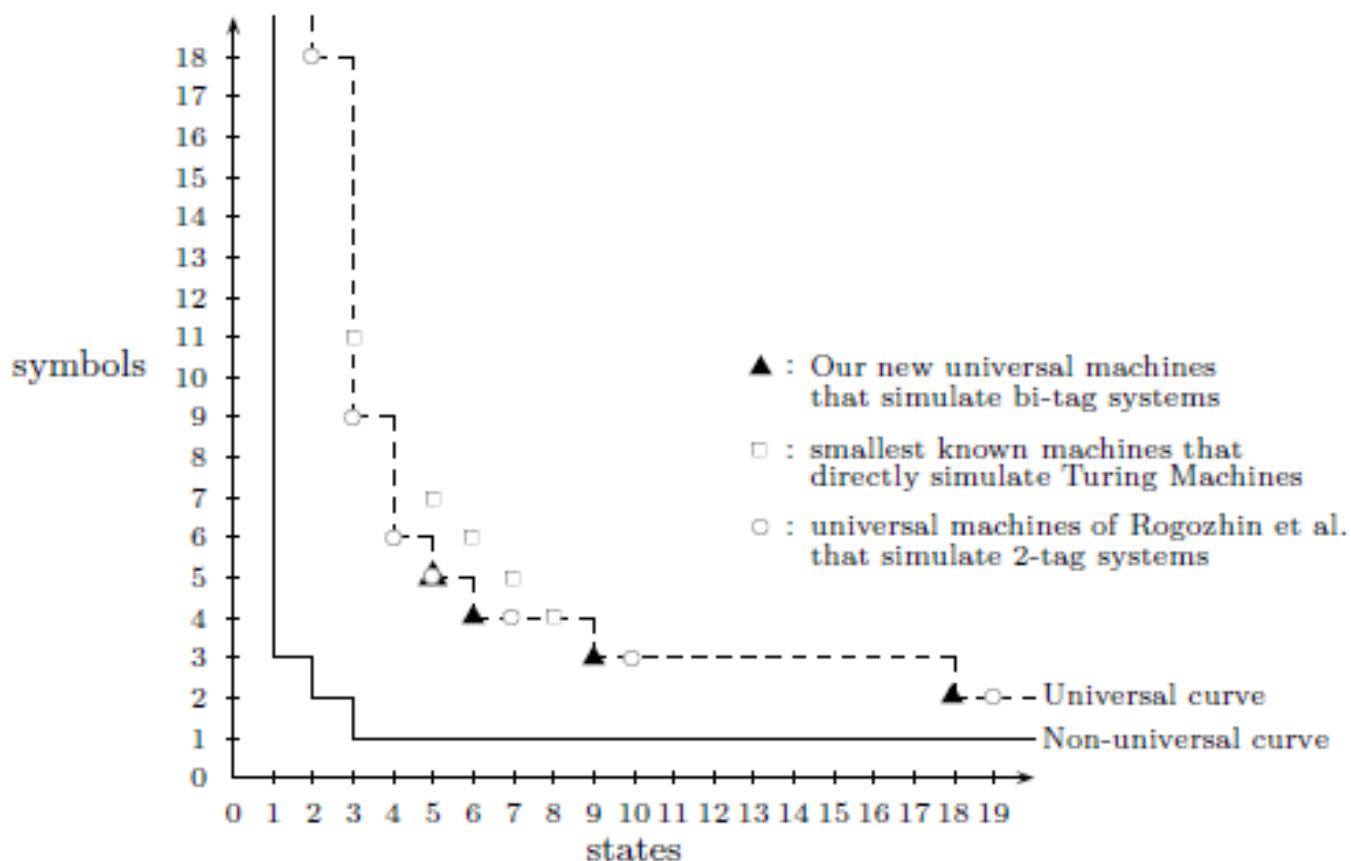


Fig. 1: Current state-symbol plot of small universal Turing machines.

- See following for more head-wall fun!!!
 - https://en.wikipedia.org/wiki/Universal_Turing_machine
 - A Business Card Universal Turing Machine: http://alvyray.com/CreativeCommons/BizCardUniversalTuringMachine_v2.2.pdf
 - 4 Small UTMs: <http://www.dna.caltech.edu/~woods/download/NearyWoodsMCU07.pdf>
 - The World's smallest UTM: <https://czyborra.com/thti/rogozhin1996.pdf>