CSE 30151 Theory of Computing: Homework 1
Math and Proof Techniques

Version 1: Jan. 16, 2018

Instructions

- Unless otherwise specified, all problems from "the book" are from Version 3. When a problem in the International Edition is different from Version 3, the problem will be listed as V3:x.yy/IE:x.zz, where x.zz is the equivalent number. When Version 2 is different, it will be listed as V3:x.yy/V2:x.zz. If either IE or V2 do not have a matching number, the problem text will be duplicated.
- You can prepare your solutions however you like (handwriting, LaTeX, etc.), but you must submit them in **legible PDF**. You can scan written solutions on the printer in the back of the classroom, or using a smartphone (with a scanner app like CamScanner). It is up to you to ensure that submissions are legible. REMEMBER THAT IF WE CAN"T READ IT OR SCAN IS CUT OFF¡ YOU DON"T GET A GRADE FOR IT.
- The problems marked as "TEAM" may be solved in a collaborative fashion with up to 2 other students. In such cases, your submission should have the word "TEAM" at the start of the problem, followed by the names of your collaborators (must be other students in this class). When such problems are graded, the first submission encountered by the grader for the team will be used for a common grade for all identified team members.
- Please give every PDF file a unique filename.
  - If you're making a complete submission (all problems), name your PDF file `netid-hw5.pdf`, where `netid` is replaced with your `NetID`.
  - If you're submitting some problems now and other problems later, name your file `netid-hw5-1-2-3.pdf`, where `1-2-3` is replaced with just the problems you are submitting now. This may be useful for team submissions.
  - If you use the same filename twice, only the most recent version will be graded.
  - The time of submission is the time the most recent file was uploaded.
- If you use LaTeX and want to draw something like a state diagram, consider using the `tikz` package. A reference document is on the website under "Assignments".
- You may also find the website http://madebyevan.com/fsm/ a useful tool for drawing state diagrams via drop and drag. It will output both .png image files and latex in the tikz format.
- Submit your PDF file in Sakai to the appropriate directory. Don't forget to click the Submit (or Resubmit) button!

Practice Problems

These problems are from the book, and most have solutions listed for them. They are listed here for you to practice on as needed and any answers you generate **should not** be submitted. You are free to discuss these with others, but you are not allowed to post solutions to any public forum.

1. Describing and working with sets: 0.1, 0.2, 0.3
2. Graphs: 0.9
3. Proofs: V3:0.10/IE:0.13, V3:0.14/IE:0.15, V3:0.15/IE:0.14

Book Exercises

These problems are found in the text book and are to be answered and submitted by each student. **If they are not marked as "TEAM," you are to solve them individually.** If they are marked as "TEAM" you may submit the same answer as the others in your team. In any case, use of solution manuals from any source or shared solutions is a violation of the ND Honor Code. You are also not allowed to show your solutions to another student not part of your TEAM.

1. (5pt) Book 0.6
2. (5pt) Book 0.7. Assume the relation is built from the set of positive integers N.
3. (5pt) Book 0.8
4. (5pt) V3:0.12/V2:0.11/IE:0.11
5. (TEAM-10pt) V3:0.11/IE:0.12 but not in V2: Let $S(n) = 1 + 2 + ...n$ and $C(n) = 1^3 + 2^3 + ... + n^3$. Prove each of the following by induction, then show $C(n) = S^2(n)$ using the two results.
   (a) $S(n) = 0.5 * n(n + 1)$
   (b) $C(n) = 0.25 * (n^4 + 2n^3 + n^2) = 0.25 * n^2(n + 1)^2$

Non-book Problems

The following problems are not found in the text book. **If they are not marked as "TEAM," you are to solve them individually.** If they are marked as "TEAM" you may submit the same answer as the others in your team. Use of any resource you used other than the text book or class notes must be cited. You are also not allowed to show your solutions to another student.

6. (5pt) Include the following statement as your answer: "I have read and understand both the ND Honor Code policy and the CSE Guide to the Honor Code as posted on the class web site. From the latter, I understand that the color for using an on-line solution manual for book problems is": (fill in color red, green, or yellow).
7. (5pt) Prove by construction that between any two consecutive powers of 2 2^n and 2^{n+1} where $n > 0$ there is at least one odd number.
8. (TEAM-10pt) Assume a bipartite graph such as introduced in the first day's lecture. Assume we have two sets of N vertexes, $M = \{m_1, ...m_N\}$ and $W = \{w_1, ...w_N\}$, and there are E edges = $\{(m_i, w_j)|m_i$ in M and w_j in W$\}$. Define at a high level (in pseudo code is fine) an algorithm that determines if there is a subset of MxW where each tuple (m_i, w_j) in the subset is an edge in E. (Hint: how many possible matchings are there, and if you picked one of them, how would you "verify" that the matching was valid). In big O notation, what is the complexity of your algorithm.

   In addition, define a heuristic that you might use that might improve performance. (Hint: consider the case where a vertex has degree 1).