From Summetria to Symmetry: the making of a revolutionary scientific concept
G. Hon and B. R. Goldstein, *New Studies in the History of Science and Technology*, Achimedes 20, Springer, 2008, 335 pages.
Review by Katherine Brading

The modern term 'symmetry' brings together a variety of disparate and over-lapping notions, some but not all of which fall under the modern group-theoretic definition now used in mathematical science. The rise of symmetry considerations to dominance in contemporary fundamental physics (both in quantum theory and in relativity) occurred in the twentieth century, and the modern group-theoretic concept has been explored by philosophers of physics in recent decades. Our scientific concepts are hard-won, forged in the heat of theoretical and practical utility: historical studies displaying some of the ways in which important scientific concepts were formed – and continue to develop – are very welcome.

Hon and Goldstein's book is, primarily, a historical study of the term 'symmetry' and the concepts associated with it up to the early 1800s. They are interested in what becomes of the Greek terms 'summetria' and 'summetros', and the concepts associated with them. In addition, they have an eye to the future: they are looking for the emergence of the modern notion of mirror symmetry. Thus, their historical narrative has two aspects: the first traces the use of the term "symmetry", while the second looks for evidence of features of the modern the concept -- specifically that of mirror symmetry -- independent of terminology.

Hon and Goldstein's historical narrative contains two trajectories originating with the Greeks, which they label the 'mathematical' and the 'aesthetic'. In mathematics, the term was used to mean (p. 70) "a measure that expresses a ratio between quantities of the same kind (having a common measure; commensuration or due proportion)", while aesthetically the term was used to mean well proportioned. These 'two trajectories' distinguished by Hon and Goldstein should not be thought of as capturing a clean separation. The 'mathematical' trajectory is covered in Chapter 2, beginning with Euclid's *Elements* where the term symmetry (asymmetry) is applied to magnitudes and is used to mean commensurable (incommensurable). By definition, Hon and Goldstein explain, magnitudes X and Y are commensurable iff they share a common measure, i.e. there exists a magnitude W such that X=mW and Y=nW, where m and n are integers. In other words, X and Y stand to one another in the relation of an integer ratio. Hon and Goldstein locate the "common measure" concept attaching to the term "symmetry" also in Archimedes, both in mathematical contexts, and also in mechanics with respect to the balance and equilibrium. For example, the thirteenth century Latin translation of Archimedes is quoted as follows (p. 75): 'Commensurable [summetra] magnitudes are in equilibrium at distances reciprocally proportional to the weights.' Commensurability was also an issue in astronomy (see p. 80): Hon and Goldstein quote from Oresme's discussion of whether the motions of the heavens are commensurable or not: do the periods of the planets have a common measure?

Both the mathematical and the aesthetic uses of the term 'symmetry' survive into the early seventeenth century, but by the mid eighteenth century the mathematical use (meaning 'commensurability') has disappeared. Moreover, Hon and Goldstein emphasize that Copernicus, Galileo, and Kepler use the term – insofar as they use it at all – exclusively in its aesthetic sense.

The aesthetic trajectory is set out in Chapters 3 and 4. It is in the aesthetic trajectory that Hon and Goldstein seek the origins of the modern concept. Their story begins with Plato and Aristotle, but the focus is on Vitruvius. For Vitruvius, 'symmetry' means that the parts stand in an appropriate relation to the whole (they are in proportion), with less emphasis being placed on any relations among the parts themselves with respect to the whole. With Alberti (1404-1472), the concept is extended to pay greater attention to the relations among the parts: specifically, the 'correspondence' between identical elements placed at an equal distance on either side of a central element. It is this notion that enters into Perrault's 1673 definition of symmetry. As others have pointed out, and as Hon and Goldstein state (p. 120), a 'decisive move towards the modern concept of symmetry was taken in France by Perrault in 1673'. They remark that (p. 121) 'Perrault proposed a precise definition that provided the ground for a new conception of symmetry in architecture. This, we argue, is one of the principal mileposts on the way to the modern scientific concept of symmetry.' That said, Hon and Goldstein insist that there are two crucial differences between 'the modern concept' and that defined by Perrault. First, they maintain that Perrault does not have *reflection*: what he has is pairs of identical elements at equal distances from a central feature. Second, they claim that the modern concept requires no such central element, and no relation of parts to the whole. I will return to these claims below. Hon and Goldstein's central thesis is that these further developments were made by Legendre, whose story is the subject of Chapter 8.

The distinguishing features of Legendre's definition (1794), according to Hon and Goldstein, are as follows. First, Legendre recognizes what is involved in reflection *explicitly*. Based on the textual evidence offered by Hon and Goldstein, this seems right. However, Hon and Goldstein make the further claim that Perrault's version does *not* include reflection even implicitly, and this is more problematic. For example, if we begin with a pair of left ears and translate one member to the right side of the head, we will end up trying to attach a left ear to the right side of someone's head; if we reflect one member of the pair then we will get a right ear and everything will work out just fine. Hon and Goldstein state (p. 155) that what is missing from Perrault's definition is that *a*symmetrical elements must be flipped (and not merely translated) by the bilateral symmetry. The grounds for this seem to be that Perrault is dealing with elements that are themselves relevantly symmetrical (rectangular windows, not ears). However, supposing that Perrault's elements are thus restricted, since a true reflection and a 'mere translation' will yield the same outcome, we won't be able to tell which operation he is thinking of. Second, in Hon and Goldstein's account they discuss how Perrault's definition applies to the human face, explicitly including ears. So far as I can tell, nothing in their text warrants the strong conclusion that Perrault's definition lacked this feature of the concept of symmetry.

Second, Legendre's definition drops the relationship of parts to whole and considers instead the relations between two solids: given that they satisfy the requirements of similarity and equality (in magnitude), there remains the question of whether they can be superposed. If they can, then they are *absolutely* similar and equal, in Legendre's terminology, and if they cannot then they have similarity and equality *by symmetry*. Hon and Goldstein consider this to be crucial. They write (p. 239): Legendre's 'new concept is unrelated to the proportionality of the parts with respect to a whole - which is the defining characteristic of the ancient concept of symmetry. With Legendre, symmetry becomes a relation between two solid figures, irrespective of their arrangement in space. Proportions of the parts with respect to a whole are meaningless in this new definition.' Thus, for Hon and Goldstein, Perrault's definition contains vestiges of the ancient concept that need to be dropped in order for "the modern concept" to emerge. This is somewhat puzzling, since the modern concept crucially retains the notion of the inter-changability of parts with respect to whole. It is also worth noting that superposability is not a brute relation between two solids, but requires their embedding in a common space such that the relevant relations are defined.

Third, whereas Perrault's definition concerns the plane, Legendre's concept applies to solids. Indeed, it applies to solids exclusively. Hon and Goldstein emphasize that according to Legendre plane figures are always absolutely similar because you can always flip them in the third dimension. With hindsight, however, Legendre draws the important conceptual boundary in the wrong place. Just as non-superposable plane figures can be rotated in a third dimension to render them superposable, so for three-dimensional solids rotated in four dimensions: the need for an additional dimension was the crucial conceptual lesson associated with superposability and reflection.

Thus, each of the 'distinguishing' features of 'the modern concept' identified in Legendre's definition by Hon and Goldstein seem to me to require a more careful discussion. Hon and Goldstein's conclusion is this (p. 260): "The difference between Legendre's definition of symmetry in solid geometry, on the one hand, and respective symmetry in architecture, on the other, is categorical." Moreover, Hon and Goldstein point to the increasing use of the term 'symmetry' in scientific texts in the period following Legendre's important study of Euclidean geometry as evidence that his definition was a crucial conceptual breakthrough.

Other places of the book that will be of interest to philosophers include Chapter 6, entitled 'The treatment of symmetry in natural history', which includes an interesting link between symmetry in botany (in plant classification) and in crystallography. Hauy's work makes use of the idea of elements that can undergo substitutions without the appearance of the crystal changing. This is very much a part of the modern group-theoretic concept. Hon and Goldstein claim that Hauy's concept of symmetry came from Vitruvius through botany, and that there is a continuous story. Chapter 7 is focussed entirely on reflection symmetry, and concerns Euler (on celestial globes) and Kant (on incongruent counterparts). In Chapter 1, Hon and Goldstein set out their case that one role for history of science is to tell the stories of the emergence of scientific concepts. For philosophers of science, such historical work is invaluable, and I for one am cheering loudly for more such work to be done. Their study provides one window on the developments that led to our modern concept of symmetry, framed by their 'two paths' analysis and their focus on mirror symmetry.

In addition to the historical narrative outlined above, the book contains a second narrative: a critique of existing historical discussions of symmetry, including the historical stories told by philosophers. This narrative is much less interesting to me, but I have two reasons for commenting on it. First, the critical narrative informs the historical narrative, and this is something of which readers should be aware. In the *Preface* Hon and Goldstein set out their thesis thus: the 'received view' is that the modern concept of symmetry has always existed, but in fact 'prior to the mid-18th century the term, symmetry, does not occur in any of its modern senses. Moreover, there was no term of expression to connote the meaning of the modern concept of symmetry.' In response to this 'received view' the authors are at pains throughout the book to stress that the various historical concepts they discuss *lack* one or more features of the modern-day concept, or differ from it in various ways. Their particular focus for a 'missing piece' is reflection symmetry. Second, it would be odd for me to end this review without commenting on some of the places where my work with Elena Castellani comes under attack. In what follows I offer some comments on this second narrative, and its impact on the first narrative.

Chapter 5 begins with Copernicus and Galileo, and stresses the use of symmetry to mean 'harmony', then moves to Kepler's and Descartes's discussions of snowflakes, and to Kepler and Leibniz's use of harmony. Goldstein and Hon show that where the term 'symmetry' is in use by these characters – which is very rarely – it is in the sense of harmony, or of being well-proportioned. Moreover, harmony is a key concept at work in Copernicus, Kepler and Leibniz. On this basis, they conclude that (p. 176) 'the usages of symmetry in science and philosophy in the early modern period have nothing to do with the modern concept of symmetry, or with bilateral symmetry and mirror images.' This conclusion is based on tracing the history of the *term* 'symmetry', but the inference seems to me to be problematic. For example, when discussing Galileo, Hon and Goldstein point out that he did not use the term 'symmetry' in the modern sense. They then go on to say: 'It is often claimed that Galileo appealed to symmetry arguments in the modern sense but, given our historical analysis, this is anachronistic because [the term] symmetry did not have the modern meaning at the time, and Galileo did not use it that way.' They then footnote Castellani and myself as an example of such 'anachronistic readings'. But the connection between Galileo and the modern concept of symmetry has nothing to do with his use of the term 'symmetry' when discussing the harmony of Copernicus's system. The reference is to Galileo's famous ship experiment, in which he shows that observations made in the cabin of a ship do not enable us to tell whether the ship is at rest or moving uniformly and 'not fluctuating this way and that'. This *does* express crucial ingredients of the modern concept, and it *is* a central part of Galileo's arguments in support of the Copernican system. Galileo's ship experiment is a hugely important chapter in the story of the formation of the modern concept of symmetry: about that there can be no reasonable dispute.

Indeed, for philosophers of physics the exclusive focus on the concept of reflection symmetry is frustrating. Reflection symmetry is a discrete symmetry, but at least as important in contemporary physics are the continuous symmetries. These are missing entirely from Hon and Goldstein's story. Just as when discussing Galileo, there is no mention of the relativity principle when discussing Newton either, nor is there mention of homogeneity and isotropy of space, and so on. Hon and Goldstein's discussion of Newton consists of a few lines restricted entirely to Newton's use of incommensurability in mathematics. Indeed, the methodology adopted by Hon and Goldstein would leave out Einstein's 1905 paper setting out special relativity; this paper is widely regarded as offering the first example of the explicit use of a symmetry principle (the relativity principle) to constrain physical theorizing, yet the term 'symmetry' does not appear with respect to the relativity principle in this paper. In short, important strands that contributed to the modern group-theoretic concept (despite the fact they were not given the *name* symmetry), find no place in Hon and Goldstein's account.

Hon and Goldstein's choice of methodology is explicit, and for further discussion of this, and their account of the 'received view' (which, by the way, I do not recognize), the reader should refer to Chapter 1. This chapter is approximately 50 pages in length, and contains a detailed overview of the story that Hon and Goldstein plan to tell, along with the methodology that they propose to use. This is also where Hon and Goldstein take to task both historians and philosophers of science for the stories they tell about the origins of the modern concept of symmetry. The chapter is not only long, but repetitive, and would have benefited from some judicious editing.

Another example of the criticisms levelled at existing discussions, for historical anachronism, occurs at the beginning of section 5.4.2. Here, Hon and Goldstein state that in our (2003) Castellani and I claim that Kepler and Leibniz were motivated by symmetry considerations, where symmetry is to be understood in the modern sense. This isn't right. In the case of Kepler (Brading and Castellani, 2003, pp. 1-2), we are explicitly discussing symmetry understood as harmony. We go on in the next paragraph to discuss 'a different notion of symmetry' which 'slowly emerged in the modern era' (*ibid*, p. 2), and discuss Perrault, but no attempt is made to impute this notion of symmetry to Kepler. Our appeal to the Leibnizian Principle of Sufficient Reason (PSR) occurs in the context of symmetry *arguments* (*ibid*, pp. 9-11): such arguments describe a scenario or situation as exhibiting a certain balance or symmetry, and draw conclusions on this basis, where the mode of inference can be understood as an application of PSR. There is no need to read the modern group-theoretic notion of symmetry (nor Leibniz's own PSR) into the historical arguments (of Archimedes, Anaximander, and so on) in order to recognize in them the above general argument form. Nor does this prevent us from reformulating the arguments with an explicit appeal to the modern concept of symmetry, especially where such reformulations prove philosophically illuminating. Indeed, as we note (*ibid*, p. 10), the first formulation of this type of argument that appeals *explicitly* to symmetry is due to Pierre Curie at the end of the nineteenth century.

I have explained why I disagree with the framework of both the historical and the critical narrative offered by Hon and Goldstein but this will, I hope, be viewed as a contribution to fruitful discussion between historians and philosophers of science. Conceptual history of science is out of fashion, and for philosophers of science it is a pleasure to read historians engaged with the technical details of mathematical science, especially pertaining to a concept that is so important for contemporary physics.